

10-1 Exponential Functions



Example #1 Chatty Chris K. likes spreading naughty rumors about students. Chris knows telling just one person the rumor will make it spread like wild-fire. If each person that hears the rumor tells two other people, how many new people will know the rumor on the 25th round of phone calls?

Write a function to represent this exponential growth, and then answer the question.

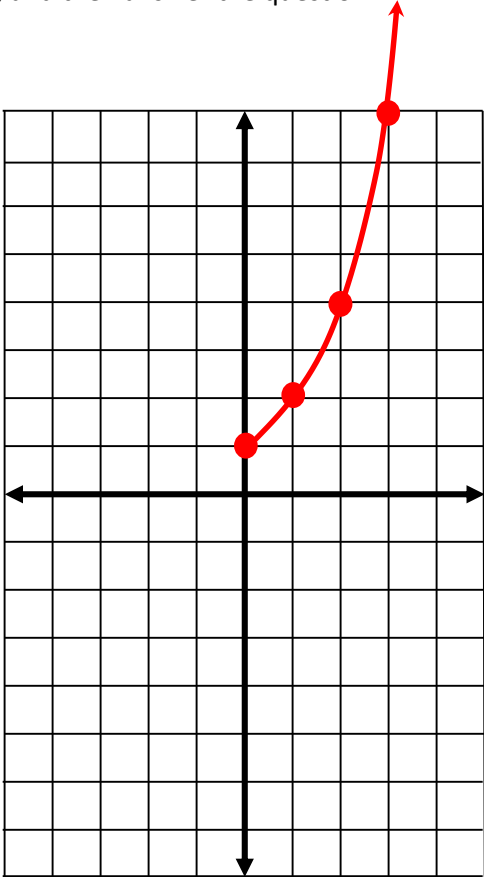
$$y = 2^x$$

$$y = 2^{25}$$

$$y = 33,554,432$$

Now graph the function.
Name its domain & range.

x	$y = 2^x$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	etc.



$$\text{Domain: } x \geq 0$$

$$\text{Range: } y \geq 1$$

Example #2 After many years of teaching (and loads of stress), Mr. Paull has determined the number of students with a legitimate brain in their head has decreased by an eighth each successive year. If Mr. Paull began teaching 21 years ago, and had 128 geniuses that first year, how many of his students have legit craniums this year?

$$y = 128(7/8)^x$$

$$\text{Domain: } x \geq 0$$

$$\text{Range: } y \leq 128$$

$$\text{Approx. 7 students}$$

Write a function, and then try to sketch its graph. Use a calculator to verify your theory.

What is the domain & range of the function?

Go to the end of the notes to see the sketch of the graph.

Was example #2 an example of exponential growth or decay?

decay

How can you tell by the equation (function) that you are seeing growth or decay?

the fraction

Can you give any examples of exponential growth or decay?

growth: bacteria, population
decay: radiation, earth's forests



WRITING EXPONENTIAL EQUATIONS

Write an exponential function for the graph that passes through the given points.

$$y_1 = y_0 r^{(x_1 - x_0)}$$

1) (0, 5) & (3, 320)

2) (0, -3) & (-4, -48)

Solve for "r" to find the rate of growth/decay.

$$\begin{aligned} 320 &= 5r^{(3-0)} & 64 &= r^3 & \text{Function} \\ 320 &= 5r^3 & 4 &= r & f(x) = 5(4)^x \end{aligned}$$

$$\begin{aligned} -48 &= -3r^{(-4-0)} & 16 &= 1/r^4 & r &= 1/2 \\ -48 &= -3r^{-4} & 16/1 &= 1/r^4 & \text{Function} \\ 16 &= r^{-4} & 1/16 &= r^4 & f(x) &= -3(1/2)^x \end{aligned}$$

SOLVING EXPONENTIAL EQUATIONS

Solve each equation.

3) $3^{2n+1} = 81$

$$\begin{aligned} 3^{2n+1} &= 3^4 \\ 2n+1 &= 4 \\ 2n &= 3 \\ n &= 3/2 \end{aligned}$$

4) $4^{2x} = 8^{x-1}$

$$\begin{aligned} (2^2)^{2x} &= (2^3)^{x-1} \\ 2^{4x} &= 2^{3x-3} \\ 4x &= 3x-3 \\ x &= -3 \end{aligned}$$

5) $\frac{1}{125} = 5^{5k+7}$

$$\begin{aligned} \frac{1}{5^3} &= 5^{5k+7} \\ 5^{-3} &= 5^{5k+7} \\ -3 &= 5k+7 \\ -10 &= 5k \\ -2 &= k \end{aligned}$$

SOLVING EXPONENTIAL INEQUALITIES

Solve each inequality.

6) $4^{3p-1} \geq \frac{1}{256}$

$$\begin{aligned} 4^{3p-1} &\geq 4^{-4} \\ 3p-1 &\geq -4 \\ 3p &\geq -3 \\ p &\geq -1 \end{aligned}$$

7) $9^{-3x} < 17^6$

$$\begin{aligned} (3^2)^{-3x} &< (3^3)^6 \\ 3^{-6x} &< 3^{18} \\ -6x &< 18 \\ x &> -3 \end{aligned}$$

Example#2 (from page 1)

