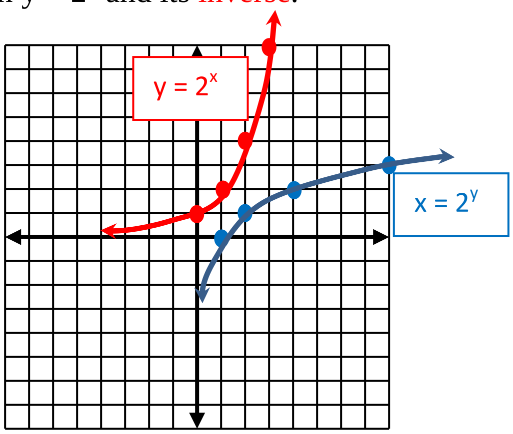
Section 10-2

LOGARITHMS AND LOG FUNCTIONS

Graph $y = 2^x$ and its inverse.



The inverse equation of $y = 2^x$ is $x = 2^y$ or y is called the ***logarithm*** of x, written $y = \log_b x$

Examples: Write each equation in exponential form.

1) $\log_8 1 = 0$ — $8^0 = 1$

2) $\log_2 \frac{1}{16} = -4$ — $2^{-4} = 1/16$

3) $\log_3 \frac{1}{27} = -3$ — $3^{-3} = 1/27$

Examples: Write each equation in logarithm form.

Think of it this way: $\log_{base} answer = exponent$

4) $10^3 = 1000$ — $\log_{10} 1000 = 3$

5) $9^{\frac{1}{2}} = 3$ — $\log_9 3 = 1/2$

6) $125^{-\frac{1}{3}} = \frac{1}{5}$ — $\log_{125} (1/5) = -1/3$

Evaluate each expression (find the exponent to which the base must be raised to).

Examples:

7) $\log_3 81$

$\log_3 81 = x$
$3^x = 81$
$3^x = 3^4 \quad x = 4$

8) $\log_4 256$

$\log_4 256 = x$
$4^x = 256$
$4^x = 4^4 \quad x = 4$

9) $\log_2 \frac{1}{32}$

$\log_2 (1/32) = x$
$2^x = 1/32$
$2^x = 1/2^5$
$2^x = 2^{-5}$
$x = -5$

The function $y = \log_b x$, where $b > 0$, and $b \neq 1$ is called a logarithmic function. The function (as shown on the earlier graph) has the following characteristics.

1) The function is continuous and one to one
2) The domain is all positive real numbers
3) The graph contains an x-intercept of 1
4) The y-axis is an asymptote of the graph
5) The range is all real numbers

Solving logarithmic equations.

Examples:

10) $\log_4 n = \frac{5}{2}$

$4^{(5/2)} = n$

$\sqrt{4^5} = n$

n = 32

11) $\log_b 121 = 2$

11) $b^2 = 121$

$b = \pm 11$, but

b>0 so b = 11

12) $\log_9 \frac{1}{3} = k$

$9^k = 1/3$

$(3^2)^k = 3^{-1}$

2k = -1

k = -1/2

Logarithmic inequalities.

Assuming the base > 0;

For >, x > answer.

For $\geq$, x $\geq$ answer.

For <, 0 < x < answer.

For $\leq$, 0 < x $\leq$ answer.

Examples:

13) $\log_5 x < 2$

less than prob.s

$0 < x < 5^2$

0 < x < 25

{ x| 0 < x < 25 }

14) $\log_4 x > 3$

greater than

$x > 4^3$

x > 64

15) $\log_2 x \leq 4$

$0 < x \leq 2^4$

$0 < x \leq 16$

{ x| $0 < x \leq 16$ }

Property of Equality

Example: If $\log_7 x = \log_7 3$, then x = 3

Examples:

16) $\log_6 (3x – 7) = \log_6 (x + 13)$

3x – 7 = x + 13

2x – 7 = 13

2x = 20   x = 10

17) $\log_5 (p^2 – 2) = \log_5 p$

$p^2 – 2 = p$

$p^2 – p – 2 = 0$

(p – 2)(p + 1) = 0

p = 2 and p = -1

18) $\log_{10} (x^2 + 3) = \log_{10} 12$

$x^2 + 3 = 12$

$x^2 = 9$

$x = \pm 3$

Inequality examples:

Property of Inequality

Example: If $\log_7 x > \log_7 3$, then x > 3

and the above and below method for number line graphing!!

19) $\log_{10} (3x – 4) < \log_{10} (x + 6)$

See the next page for #19 & 20

20) $\log_5 (2x + 1) \geq \log_5 (x – 4)$

19) $3x - 4 < x + 6$

$2x - 4 < 6$

$2x < 10$

$x < 5$

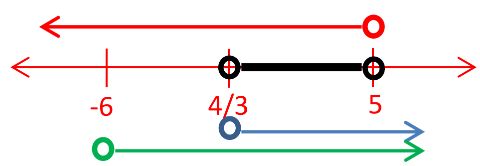
However, since the bases are non-negative, this means there are no negative results. Therefore, we must exclude all values of "x" that would make the solution negative.

Or… $3x - 4 > 0$ and $x + 6 > 0$

$3x > 4$ $\quad$ $x > -6$

$x > 4/3$

Use a number line graph (above&below) to find a final solution set.



So, the solution set is:

{ x| 4/3 < x < 5 }

20) $2x + 1 \geq x - 4$ and $2x + 1 > 0$ and $x - 4 > 0$

$x + 1 \geq -4$ $\quad$ $2x > -1$ $\quad$ $x > 4$

$x \geq -5$ $\quad$ $x > -1/2$

If needed, use the number line graph (above & below) to arrive at the following solution set:

{ x| x > 4 }