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|--|---|---|--|
| Inverse Property of Logs<br>$\log_5 5^3 = 3$ | Product Property<br>$\log_2 5x = \log_2 5 + \log_2 x$ | Quotient Property<br>$\log_4 \frac{x^2}{6} = \log_4 x^2 - \log_4 6$ | Power Property<br>$\log_b x^3 = 3\log_b x$ |
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Rewrite the following expressions using the above properties.

- 1)  $\log_9 9^{2x} = \underline{2x}$       2)  $\log_7 12^x = \underline{x \log_7 12}$       3)  $\log_6 5g - \log_6 10 = \underline{\log_6 \frac{g}{2}}$
- 4)  $\log_2 (17 \cdot 13) = \underline{\log_2 17 + \log_2 13}$       5)  $\log_8 \frac{x}{3x-1} = \underline{\log_8 x - \log_8 (3x-1)}$
- 6)  $5\log_3 y = \underline{\log_3 y^5}$       7)  $\log_x x^{(y+2)} = \underline{y+2}$       8)  $\log_7 a^2 + \log_7 25 = \underline{\log_7 25a^2}$

Solve the following equations by applying the above properties.

- 9)  $\log_{10} 27 = 3\log_{10} x$       10)  $\log_5 y - \log_5 8 = \log_5 9$       11)  $\log_9 4 + 2\log_9 5 = \log_9 w$

$$\begin{aligned} \log_{10} 27 &= \log_{10} x^3 \\ 27 &= x^3 \\ 3 &= x \end{aligned}$$

$$\begin{aligned} \log_5 \frac{y}{8} &= \log_5 9 \\ \frac{y}{8} &= 9 \\ y &= 72 \end{aligned}$$

$$\begin{aligned} \log_9 4 + \log_9 5^2 &= \log_9 w \\ \log_9 4 + \log_9 25 &= \log_9 w \\ \log_9 (4 \cdot 25) &= \log_9 w \\ \log_9 100 &= \log_9 w \\ 100 &= w \end{aligned}$$

- 12)  $\log_{10} x + \log_{10} (3x - 5) = \log_{10} 2$       13)  $\log_4 (n + 1) - \log_4 (n - 2) = 1$

$$\begin{aligned} \log_{10} x(3x - 5) &= \log_{10} 2 \\ x(3x - 5) &= 2 \\ 3x^2 - 5x &= 2 \\ 3x^2 - 5x - 2 &= 0 \\ \text{run program} \\ x &= 2, -1/3, \text{ but } -1/3 \text{ is undefined} \end{aligned}$$

$$\begin{aligned} \log_4 \frac{n+1}{n-2} &= 1 & 4n - 8 &= n + 1 \\ 4^1 &= \frac{n+1}{n-2} & 3n &= 9 \\ 4(n-2) &= n+1 & n &= 3 \end{aligned}$$

- 14)  $\log_3 d + \log_3 3 = 3$       15)  $\log_2 x + 2\log_2 5 = 0$       16)  $3\log_4 y = 6$

$$\begin{aligned} \log_3 3d &= 3 \\ 3^3 &= 3d \\ 27 &= 3d \\ 9 &= d \end{aligned}$$

$$\begin{aligned} \log_2 x + \log_2 5^2 &= 0 \\ \log_2 x + \log_2 25 &= 0 \\ \log_2 25x &= 0 \\ 2^0 &= 25x \\ 1 &= 25x & 1/25 &= x \end{aligned}$$

$$\begin{aligned} \log_4 y^3 &= 6 \\ 4^6 &= y^3 \\ 4096 &= y^3 \\ 16 &= y \end{aligned}$$

Use the change of base formula to approximate the following values to 4-decimals.

$$\log_a n = \frac{\log_{10} n}{\log_{10} a}$$

17)  $\log_5 12$

$$= \frac{\log 12}{\log 5} \approx 1.5440$$

18)  $\log_8 32$

$$= \frac{\log 32}{\log 8} \approx 1.6667$$

19)  $\log_{11} 9$

$$= \frac{\log 9}{\log 11} \approx 0.9163$$

20)  $\log_7 \sqrt{8}$

$$= \frac{\log \sqrt{8}}{\log 7} \approx 0.5343$$

21)  $\log_6 \frac{3}{4}$

$$= \frac{\log 3/4}{\log 6} \approx -0.1606$$

Solve each equation or inequality using common logs ( $\log_{10}$ ). Round all answers to four decimals.

22)  $9^m \geq 100$

$$\begin{aligned} \log 9^m &\geq \log 100 \\ m \log 9 &\geq \log 100 \\ m &\geq \frac{\log 100}{\log 9} \\ m &\geq 2.0959 \end{aligned}$$

23)  $27 = 4^{2x}$

$$\begin{aligned} \log 27 &= \log 4^{2x} \\ \log 27 &= 2x \log 4 \\ \frac{\log 27}{2 \log 4} &= x \\ 1.1887 &\approx x \end{aligned}$$

24)  $9^{z-2} > 38$

$$\begin{aligned} \log 9^{z-2} &> \log 38 \\ (z-2) \log 9 &> \log 38 \\ z-2 &> \frac{\log 38}{\log 9} \\ z &> \frac{\log 38}{\log 9} + 2 \\ z &> 3.6555 \end{aligned}$$

25)  $5^{x^2-3} = 72$

$$\begin{aligned} \log 5^{x^2-3} &= \log 72 \\ (x^2-3) \log 5 &= \log 72 \\ x^2-3 &= \frac{\log 72}{\log 5} \\ x^2 &= \frac{\log 72}{\log 5} + 3 \\ x &= \sqrt{\frac{\log 72}{\log 5} + 3} \\ x &\approx \pm 2.3785 \end{aligned}$$

26)  $4^{2x} = 9^{x+1}$  \*hint: this one will require you to use a GCF at some point.

$$\begin{aligned} \log 4^{2x} &= \log 9^{x+1} \\ 2x \log 4 &= (x+1) \log 9 \\ 2x \log 4 &= x \log 9 + \log 9 \\ 2x \log 4 - x \log 9 &= \log 9 \\ x(2 \log 4 - \log 9) &= \log 9 \\ x &= \frac{\log 9}{2 \log 4 - \log 9} \\ x &\approx 3.8188 \end{aligned}$$

The key to success is to know when to use which property. # in front of a log, use the power prop. + sign between two logs, use the product property. - sign between two logs, use the quotient property. Single log with no variables, use the change of base formula. No logs, with crazy looking exponents, apply the common log to both sides.

