

SECTION 10-4 COMMON LOGS



The pH level of a substance is given by the formula: $\text{pH} = -\log_{10} H^+$ where H^+ is the substance's hydrogen ion concentration. If the hydrogen concentration of sauerkraut is 2987×10^{-4} , what is its pH level?



$\text{pH} = -\log_{10} (2987 \times 10^{-4})$ Good news! Your calculator will do base₁₀ logs. It's called "common log" and is literally the LOG button on your calculator!
 $\text{pH} = -\log(2987 \times 10^{-4})$
 $\text{pH} \approx 3.5$

Solving exponential equations using common logarithms.

Examples: 1) $3^x = 11$ If the idea is to get x by itself, how do I get rid of the 3?

$$\begin{aligned} 3^x &= 11 \\ \log 3^x &= \log 11 \\ x \log 3 &= \log 11 \\ x &= \frac{\log 11}{\log 3} \end{aligned}$$

simply pop it into your calc. and bam...
 $x \approx 2.1828$

Answer: take the common log of both sides. Since base₁₀ is the only "common" log, it is unnecessary to include the ₁₀ when writing it.

2) $6^x = 42$

$$\begin{aligned} \log 6^x &= \log 42 \\ x \log 6 &= \log 42 \\ x &= \frac{\log 42}{\log 6} \\ x &\approx 2.086 \end{aligned}$$

3) $5^{3y} < 8^{y-1}$

$$\begin{aligned} \log 5^{3y} &< \log 8^{y-1} & y(3\log 5 - \log 8) &< -\log 8 \\ 3y \log 5 &< (y-1)\log 8 & y &< \frac{-\log 8}{3\log 5 - \log 8} \\ 3y \log 5 &< y \log 8 - \log 8 & y &< -0.7564 \\ 3y \log 5 - y \log 8 &< -\log 8 & & \text{keep in mind it is an } \approx \end{aligned}$$

CHANGE OF BASE FORMULA

$$\log_a n = \frac{\log_b n}{\log_b a}$$

where a, b and n are positive numbers (a, b ≠ 1)

Question? If you can use any base you want, what should we use?

Examples:

Approximate all answers to 4-decimal places.

Answer: base₁₀ (common log) because we can use the calculator!!

4) $\log_4 25$

$$\log_4 25 = \frac{\log 25}{\log 4} \approx 2.3219$$

5) $\log_6 8$

$$\log_6 8 = \frac{\log 8}{\log 6} \approx 1.1606$$

6) $\log_6 9^{w-5} = 2$

$$\begin{aligned} (w-5)\log_6 9 &= 2 \\ (w-5)\frac{\log 9}{\log 6} &= 2 \\ w-5 &= 2 \cdot \frac{\log 6}{\log 9} \\ w &= 2 \cdot \frac{\log 6}{\log 9} + 5 \\ w &\approx 6.6309 \end{aligned}$$