Algebra II

FACTORING

In my humble opinion, there are
8-different varieties of factoring to be concerned with.

TRINOMIAL FACTORING

Warm up: complete the FOIL problem: $(x - 7)(3x + 8) =$ $3x^2 + 8x - 21x - 56$ $= 3x^2 - 13x - 56$

Now go backwards: $(x + 9)(x + 8) = x^2 + 17x + 72$

mult. to 72 & adds up to 17

Examples: 1) $y^2 - 11y + 30$ 2) $w^2 + 3w - 40$ 3) $r^2 - 5rq - 36q^2$

$(y - 6)(y - 5)$ $(w - 5)(w + 8)$ $(r - 9q)(r + 4q)$

To check: just re-FOIL using only the O & I
O = -5y, I = -6y, combine to get -11y (checks)

DIFFERENCE OF 2-SQUARES

Warm up: FOIL, or more appropriately FL, this problem: $(2n + 9)(2n - 9) =$ $4n^2 - 81$

Now go backwards: $(x + 7)(x - 7) = x^2 - 49$

Examples: 1) $m^2 - 121$ 2) $25j^2 - 9$ 3) $c^6 - \frac{25}{64}$

$(m + 11)(m - 11)$ $(5j + 3)(5j - 3)$ $\left(c^3 + \frac{5}{8}\right)\left(c^3 - \frac{5}{8}\right)$

PERFECT SQUARES

Treat these just like the previous examples, and see what makes them unique.

Examples: 1) $a^2 + 12a + 36$ 2) $m^2 - 4mn + 4n^2$ 3) $9x^2 - 24x + 16$

$(a + 6)(a + 6)$ $(m + 2n)(m + 2n)$ $(3x + 4)(3x + 4)$

$=(a + 6)^2$ $=(m + 2n)^2$ $=(3x + 4)^2$

For perfect squares, simplest form is to
write $(\quad)^2$

GREATEST COMMON FACTOR (GCF)

is the most forgotten, #1 mistake of all the types of factoring, so PAY ATTENTION!

Warm up: use distributive prop. to simplify: $3x^3y(5x^2 - 7y) =$ $15x^5y - 21x^3y^2$

Now go backwards: $5bc^2$(3 + bc) = $15bc^2 + 5b^2c^3$

Examples:

1) $9x - 12xy + 3x^2y$ — $3x(3 - 4y + xy)$

2) $8m + 6n + 10$ — $2(4m + 3n + 5)$

3) $-14r^3 - 21r^2 - 7r$ — $-7r(2r^2 + 3r + 1)$

TRINOMIAL (a > 1)

Comparison: $x^2 + 13x + 40$ $= (x + 8)(x + 5)$

why 8 & 5?

because 8 & 5 add up to 13

$3x^2 + 23x + 40$ = ? ? ? ?

why won't anything add up to 23?

because of the 3 in $3x^2$, if you were to FOIL the answer back out, the 3 will change the outcome

Examples:

1) $3x^2 + 23x + 40$

possible ways to get 40: 1&40, 2&20, 4&10, 5&8
Use trial & error to find the one that checks to 23

$=(3x + 8)(x + 5)$

Check the O & I from FOIL: $(3x)(5) = 15x$
$(8)(x) = 8x$, combine to get $23x$. Checks!

2) $8m^2 + 47m - 6$

$=(8m - 1)(m + 6)$

3) $12y^2 + 19y - 18$

$=(3y - 2)(4y + 9)$

Check the O & I from FOIL: $(3y)(9) = 27y$
$(-2)(4y) = -8y$, combine to get $19y$. Checks!

PERFECT CUBES

Just for kicks, attempt the following. $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$
Notice the size of the parenthesis.

Example:	$y^3 + 125$	step(1), treat it like FOIL	(y)(y^2) (y 5)(y^2 25)
		step(2), the middle; mult. what's in the first ( )	(y 5)(y^2 5y 25)
		step(3), clean with SOAP	$(y + 5)(y^2 - 5y + 25)$

Examples:

1) $g^3 + 64$ — $(g + 4)(g^2 - 4g + 16)$

2) $27x^3 - 1$ — $(3x - 1)(9x^2 + 3x + 1)$

3) $8a^3 - \frac{1}{27}b^3$ — $\left(2a - \frac{1}{3}b\right)\left(4a^2 + \frac{3}{3}ab + \frac{1}{9}b^2\right)$

MULTIPLE-STEP FACTORING

Warm up: Factor (1-step only): $4x^3 - 2x + 12$ = prime, can't do it!

GCF's are almost always the first step, so if you forget to ***always*** attempt a GCF first…

Examples:

1) $x^3 + 5x^2 - 14x$

GCF: $x(x^2 + 5x - 14)$
factor again w/trinomial factoring
$x^2 + 5x - 14 = (x + 7)(x - 2)$

Answer: $x(x + 7)(x - 2)$

2) $8h^2 - 18$

GCF: $2(h^2 - 9)$
factor again w/difference of 2-squares
$= 2(h + 3)(h - 3)$

3) $y^2 + w^3y^2$

GCF: $y^2(1 + w^3)$
factor again w/perfect cube
$= y^2(1 + w)(1 - w + w^2)$

4) $ax^2 - 8ax + 16a$

GCF: $a(x^2 - 8x + 16)$
now use perfect square
$= a(x - 4)^2$

5) $15p^2 - 35p - 30$

GCF: $5(3p^2 - 7p - 6)$
now use trinomial (a > 1)
$= 5(3p + 2)(p - 3)$

6) $n^6 - 1$

No GCF! use diff of 2-squares
$(n^3 + 1)(n^3 - 1)$
now use perfect cube on both
$(n + 1)(n^2 - n + 1)(n - 1)(n^2 + n + 1)$

To 2nd step or not to 2nd step, that is the question.

Examples:

1) $3j^7 - 9j^2 + 15j^3$

2) $8mn^2 + 8mn + 2m$

3) $kx - 4k$

GCF = $3j^2 (3j^5 - 3 + 5j)$	GCF = $2m (4n^2 + 4n + 1)$	GCF = $k (x - 4)$
When did we ever split up a j^5? Never!	There is an n-"squared" left in the ( ). 2nd step!!!	How can we possibly split the x up if there is only one? Can't!
DONE	$2m (2n + 1)(2n + 1)$ $2m (2n + 1)^2$	DONE