Section 6-2

# FACTOR BY GROUPING

Recognition: factor by grouping can be identified by polynomials with even terms ( 4, 6, etc. parts). It will be apparent that there are GCF's among them.

Examples:

The ( ) must match!
Otherwise, try a different pair

1) Factor $a^3 - 4a^2 + 3a - 12$

Pick pairs that have a GCF:
$a^3 - 4a^2$ $\quad$ $3a - 12$
$= a^2(a - 4)$ $\quad$ $= 3(a - 4)$
$= (a - 4)(a^2 + 3)$

2) Factor $x^2 + 6y^3 + 3xy + 2xy^2$

Pick pairs that have a GCF:
$x^2 + 3xy$ + $6y^3 + 2xy^2$
$x(x + 3y)$ + $2y^2(3y + x)$
$= (x + 3y)(x + 2y^2)$

3) Factor $3x^2 - 12 + 4x - x^3 - 4w + wx^2$

$3x^2 - 12$ + $4x - x^3$ + $-4w + wx^2$
$3(x^2 - 4)$ + $x(4 - x^2)$ + $w(-4 + x^2)$
$3(x^2 - 4)$ – $x(x^2 - 4)$ + $w(x^2 - 4)$
$= (x^2 - 4)(3 - x + w)$
$=(x + 2)(x - 2)(3 - x + w)$

4) Factor $2b - 2 + a - ab$

$2b - 2$ + $a - ab$
$2(b - 1)$ + $a(1 - b)$
$2(b - 1)$ – $a(b - 1)$
$(b - 1)(2 - a)$

Switch sign in front of ( ), switches signs inside of ( )

# SOLVING HIGHER DEGREE EQUATIONS BY FACTORING

Examples:

1) By GCF

$2x^5 - 32x^3 = 0$

$2x^3(x^2 - 16) = 0$
$2x^3(x + 4)(x - 4) = 0$

$x = 0,\ x = -4,\ x = 4$

2) By Trinomial factoring

$x^4 - 13x^2 + 40 = 0$

$(x^2 - 8)(x^2 - 5) = 0$
$x^2 - 8 = 0$ and $x^2 - 5 = 0$
$x^2 = 8$ $\quad$ $x^2 = 5$
$x = \pm\sqrt{8}$ $\quad$ $x = \pm\sqrt{5}$
$x = \pm 2\sqrt{2}$ $\quad$ $x = \pm\sqrt{5}$

3) By Perfect cube factoring

$27x^3 + 8 = 0$

$(3x + 2)(9x^2 - 6x + 4) = 0$
use quadratic formula
$x = -\frac{2}{3}$ $\quad$ $x = \frac{1 \pm i\sqrt{3}}{3}$

4) By ???

$x^5 + 3x^2 - 18 = 0$

can't (for now). It cannot be factored with a GCF, and we can only use tri-factoring if the second exponent is half of the first!

When you can and when you can't??? Examples:

1) can't $10x^8 + 3x^6 - 1 = 0$

2) can $x^6 - 11x^3 + 18 = 0$

3) can $4x^7 + 8x^5 = 0$

3) can $3x + x^{\frac{1}{2}} + 2 = 0$

4) can $x^{\frac{3}{4}} - 5x^{\frac{3}{8}} - 6 = 0$

4) can't $8x^{\frac{1}{3}} + 10x^{\frac{1}{9}} = 1$

MORE EXAMPLES:

First determine how many solutions (roots or zeros) you are looking for, then…
Solve each polynomial equation. **BEWARE** of multiple step problems.

1) $x^4 = 64x$

$x^4 - 64x = 0$

GCF! $x(x^3 - 64) = 0$

$x(x - 4)(x^2 + 4x + 16) = 0$

quadratic formula

$x = 0,\ x = 4,\ x = -2 \pm 2\sqrt{3}$

2) $12x^3 - 8x^2 + 3x - 2 = 0$

by grouping…

$4x^2(3x - 2) + 1(3x - 2) = 0$

$(3x - 2)(4x^2 + 1) = 0$

$4x^2 + 1 = 0$

$4x^2 = -1$

$x^2 = -1/4$

$x = \sqrt{-\frac{1}{4}}$

$x = \frac{2}{3}$ $\quad x = \pm\frac{1}{2}i$

3) $y^4 - 5y^2 = 36$

$y^4 - 5y^2 - 36 = 0$

$(y^2 - 9)(y^2 + 4) = 0$

$(y + 3)(y - 3)(y^2 + 4)$

$y^2 + 4 = 0$

$y^2 = -4$

$y = \sqrt{-4}$

$y = \pm 2i$

$y = -3,\ y = 3,$