Section 6-5

RATIONAL ZERO THEOREM

The Rational Zero Theorem provides a way to create a list of potential solutions to any polynomial function. Once the list is created, synthetic division may be used on a trial and error basis to find a correct root (or zero).

Example: $f(x) = 2x^5 + x^4 - 5x^2 - 11x + 12$

The list: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \pm\frac{3}{2}$

q-factor (factors of)

p-factor (factors of)

MINDING YOUR p's AND q's

List all the possible p-factors, q-factors, the $\frac{p}{q}$-factors for each function. Do not solve.

1) $f(x) = x^3 + 17x^2 - x - 20$

p - ±1, 2, 4, 5, 10, 20

q - ±1

$\frac{p}{q}$ - ±1, 2, 4, 5, 10, 20

2) $g(x) = 4x^4 + 2x^3 + 7x^2 - 9x + 10$

p - ±1, 2, 5, 10

q - ±1, 2, 4

$\frac{p}{q}$ - ±1, 2, 5, 10, 1/2, 5/2, 1/4, 5/4

EXAMPLES FOR COLLEGE PREP ALGEBRA II

SOLVING POLYNOMIAL EQUATIONS (by creating a list of possible rational zeros)

1) $f(x) = x^3 + 6x^2 + 4x - 15$

2) $f(x) = 2x^4 - 13x^3 + 23x^2 - 52x + 60$

See bottom of next page

3) $g(x) = x^4 + 5x^2 - 36$

4) $h(x) = 8x^4 + 46x^3 + 59x^2 - 24x - 9$

Homework: pg 371-372 1-6, 10-15, 20, 28

EXAMPLES FOR ALGEBRA II

SOLVING POLYNOMIAL EQUATIONS (by creating a list of possible rational zeros)

1) $f(x) = 2x^3 + x^2 - 13x + 6$

2) $f(x) = x^4 + 5x^2 - 36$

3) $g(x) = x^4 + x^3 - 13x^2 - x + 12$

4) $f(x) = x^4 + 6x^3 + 13x^2 + 24x + 36$

Homework: pg 371-372 3-6, 20, 28

1) p: ±1,2,6

q: ±1,2

$\frac{p}{q}$: ±1,2,6,1/2,3

2	2	1	-13	6
		4	10	-6
	2	5	-3	0

$2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

x = 1/2, -3, and 2

2) p: ±1,2,3,4,6,9,12,18,36

q: ±1

$\frac{p}{q}$: ±1,2,3,4,6,9,12,18,36

-2	1	0	5	0	-36
		-2	4	-18	36
	1	-2	9	-18	0

$1x^3 - 2x^2 + 9x - 18 = 0$

$x^2(x - 2) + 9(x - 2) = 0$

$(x^2 + 9)(x - 2) = 0$ **x = ±3i, 2, -2**

3) p: ±1,2,3,4,6,12

q: ±1

$\frac{p}{q}$: ±1,2,3,4,6,12

1	1	1	-13	-1	12
		1	2	-11	-12
-4	1	2	-11	-12	0
		-4	8	12	
	1	-2	-3	0	

$1x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

x = 3, -1, 1, -4

4) p: ±1,2,3,4,6,9,12,18,36

q: ±1

$\frac{p}{q}$: ±1,2,3,4,6,9,12,18,36

-3	1	6	13	24	36
		-3	-9	-12	-36
-3	1	3	4	12	0
		-3	0	-12	
	1	0	4	0	

$x^2 + 4 = 0$

$x^2 = -4$

x = ±2i, -3

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1) p: ±1,3,5,15

q: ±1

$\frac{p}{q}$: ±1,3,5,15

-3	1	6	4	-15
		-3	**-9**	**15**
	1	3	-5	0

$1x^2 + 3x - 5 = 0$

Use quadratic formula program

x = 1.2, -4.2, -3

2) p: ±1,2,3,4,5,6,10,12,15,20,30,60

q: ±1,2

$\frac{p}{q}$: ±1,2,3,4,5,6,10,12,15,20,30,60,

1/2,3/2,5/2,15/2

5	**2**	**-13**	**23**	**-52**	**60**
		10	-15	40	-60
	2	-3	8	-12	0

$2x^3 - 3x^2 + 8x - 12 = 0$

$2x^3 + 8x - 3x^2 - 12 = 0$

$2x(x^2 + 4) - 3(x^2 + 4)$

$(2x - 3)(x^2 + 4)$

x = 3/2, ±2i, 5

3) p: ±1,2,3,4,6,9,12,18,36

q: ±1

$\frac{p}{q}$: ±1,2,3,4,6,9,12,18,36

2					
	1	0	5	0	-36
		2	4	18	36
	1	2	9	18	0

$x^3 + 2x^2 + 9x + 18 = 0$

$x^2(x + 2) + 9(x + 2) = 0$

$(x + 2)(x^2 + 9) = 0$

x = -2, ±3i, 2

4) p: ±1,3,9

q: ±1,2,4,8

$\frac{p}{q}$: ±1,3,9,

1/2,3/2,9/2,1/4,3/4,9/4,1/8,3/8,9/8

-3	8	46	59	-24	-9
		-24	-66	21	9
-3	8	22	-7	-3	0
		-24	6	3	
	8	-2	-1	0	

$8x^2 - 2x - 1 = 0$

Use quadratic formula program

x = .5, -.3, -3