

SECTION 7-1

OPERATIONS ON FUNCTIONS



The following should be review...

Use the two functions shown to compute: $(f + g)(x)$ $(f - g)(x)$ $(f \cdot g)(x)$ $\left(\frac{f}{g}\right)(x)$

$$f(x) = 2x^2 - 7x - 4 \quad \& \quad g(x) = x - 6$$

$(f+g)$ $= (2x^2 - 7x - 4) + (x - 6)$ <p>combine like terms</p> $= 2x^2 - 6x - 10$	$(f-g)$ $= (2x^2 - 7x - 4) - (x - 6)$ <p>distribute the neg. sign</p> $= 2x^2 - 7x - 10 - x + 6$ $= 2x^2 - 8x - 4$	$(f \cdot g)$ <p>Box:</p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td style="padding: 2px 10px;">$2x^2$</td> <td style="padding: 2px 10px;">$-7x$</td> <td style="padding: 2px 10px;">-4</td> </tr> <tr> <td style="padding: 2px 10px;">x</td> <td style="padding: 2px 10px;">$2x^3$</td> <td style="padding: 2px 10px;">$-7x^2$</td> </tr> <tr> <td style="padding: 2px 10px;">-6</td> <td style="padding: 2px 10px;">$-12x^2$</td> <td style="padding: 2px 10px;">$42x$</td> </tr> <tr> <td></td> <td style="padding: 2px 10px;">24</td> <td></td> </tr> </table> $= 2x^3 - 19x^2 + 38x + 24$	$2x^2$	$-7x$	-4	x	$2x^3$	$-7x^2$	-6	$-12x^2$	$42x$		24	
$2x^2$	$-7x$	-4												
x	$2x^3$	$-7x^2$												
-6	$-12x^2$	$42x$												
	24													

We are leaving division out for reasons I won't bother you with.

The following should not be review...



Tyrone has \$180 deducted from every paycheck for retirement. He can have these deductions taken before or after taxes are applied to his taxable income. His federal income tax rate is 18%.

Let x = Tyrone's pay

Write a function $r(x)$ that represents his pay less retirement.

$$r(x) = x - 180$$

Write a function $t(x)$ that represents his pay less taxes.

$$t(x) = x - 0.18x$$

If Tyrone earns \$2200 every pay period, find the difference in his net income if he has the retirement deduction taken before or after taxes.

Retirement first: $t(r(2200))$

1) find $r(2200) = 2200 - 180$
 $= \$2020$

2) find $t(2020) = 2020 - 0.18(2020)$
 $= 2020 - 363.69$
 $= \$1656.40$

Taxes first: $r(t(2200))$

1) find $t(2200) = 2200 - 0.18(2200)$
 $= 2200 - 396$
 $= \$1804$

2) find $r(1804) = 1804 - 180$
 $= \$1624$

$1656.40 - 1624 = \$32.40$ Tyrone would receive \$32.40 more in net pay if he had his retirement taken out before taxes.

COMPOSITION OF FUNCTIONS

Suppose $f(x) = 8 - 9x$, $g(x) = 2x^2 - 1$ and $h(x) = x - x^2$

Find the following: 1) $(f \circ g)(3)$

$$\begin{aligned} g(3) &= 2(3)^2 - 1 \\ &= 2(9) - 1 \\ &= 18 - 1 \\ &= 17 \\ f(17) &= 8 - 9(17) \\ &= 8 - 153 \\ &= -145 \\ (f \circ g)(3) &= -145 \end{aligned}$$

2) $h(f(2))$

$$\begin{aligned} f(2) &= 8 - 9(2) \\ &= 8 - 18 \\ &= -10 \\ h(-10) &= -10 - (-10)^2 \\ &= -10 - 100 \\ &= -110 \\ h(f(2)) &= -110 \end{aligned}$$

3) $g[f(h(1))]$

$$\begin{aligned} h(1) &= 1 - 1^2 \\ &= 1 - 1 \\ &= 0 \\ f(0) &= 8 - 9(0) \\ &= 8 - 0 \\ &= 8 \\ g(8) &= 2(8)^2 - 1 \\ &= 2(64) - 1 \\ &= 128 - 1 \\ &= 127 \\ g[f(h(1))] &= 127 \end{aligned}$$

4) $[f \circ g](x)$

$$\begin{aligned} &\text{Plug entire } g\text{-function into } f \\ &= 8 - 9(2x^2 - 1) \\ &\text{simplify} \\ &= 8 - 18x^2 + 9 \\ &= -18x^2 + 17 \\ [f \circ g](x) &= -18x^2 + 17 \end{aligned}$$

5) $g(f(x))$

$$\begin{aligned} &\text{Plug entire } f\text{-function into } g \\ &= 2(8 - 9x)^2 - 1 \\ &\text{simplify (FOIL)} \\ &= 2(8 - 9x)(8 - 9x) - 1 \\ &= 2(64 - 72x - 72x + 81x^2) - 1 \\ &= 2(64 - 144x + 81x^2) - 1 \\ &= 128 - 288x + 162x^2 - 1 \\ &= 162x^2 - 288x + 127 \\ g(f(x)) &= 162x^2 - 288x + 127 \end{aligned}$$

DOES A COMPOSITION OF FUNCTIONS EXIST?

if so, find its domain and range.



Suppose $f = \{ (7, 8), (5, 3), (9, 8), (11, 4) \}$ and $g = \{ (5, 7), (3, 5), (7, 9), (9, 11) \}$

To find $f \circ g$, start with the x-coordinate in the "g" function (5), link it to its y-coordinate (7), find the matching x-coordinate in the "f" function (7), and link it to its y-coordinate (8). i.e. (5, 8)

Find $f \circ g$ and $g \circ f$

$$\begin{aligned} &f \circ g \\ &\{(5, 8), (3, 3), (7, 8), (9, 4)\} \end{aligned}$$

$$\begin{aligned} &g \circ f \\ &\{(7, ?), (5, 5), (9, ?), (11, ?)\} \\ &\text{since } (5, 5) \text{ is the only set of} \\ &\text{coordinates that exists,} \\ &g \circ f = \{(5, 5)\} \end{aligned}$$