SECTION 8-3

Operations with Radical Expressions

Treat it like: $\sqrt[2]{b^3}$
2 goes into 3 once (outside) with a remainder of 1 (inside)

For square roots, cubed roots, etc. that do not come out even; break out the **factor trees**!

For multiplication, multiply outside X outside, and inside X inside. Simplify either before or after.

Examples: (Simplify)

1) $\sqrt{50j^4}$

$\sqrt{25}\bullet\sqrt{2}\bullet\sqrt{j^4}$

$5j^2\sqrt{2}$

2) $\sqrt[3]{40x^6y^3}$

$\sqrt[3]{8}\bullet\sqrt[3]{5}\bullet\sqrt[3]{x^6}\bullet\sqrt[3]{y^3}$

$2x^2y\sqrt[3]{5}$

3) $\sqrt{63a^6b^3}$

$\sqrt{9}\bullet\sqrt{7}\bullet\sqrt{a^6}\sqrt{b^3}$

$3a^3b\sqrt{7b}$

4) $\dfrac{\sqrt[3]{88}}{\sqrt[3]{11}}$

$=\sqrt[3]{\dfrac{88}{11}} \quad =\sqrt[3]{8} \quad =2$

5) $\sqrt[4]{\dfrac{1}{81}}$

$=\dfrac{\sqrt[4]{1}}{\sqrt[4]{81}} \quad =\dfrac{1}{3}$

6) $\sqrt{\dfrac{2r^2q^3}{49}}$

$=\dfrac{\sqrt{2r^2q^3}}{\sqrt{49}} \quad =\dfrac{rq\sqrt{2q}}{7}$

Examples: (Multiply)

7) $3\sqrt[3]{54z^7}$

$3\bullet\sqrt[3]{27z^6}\bullet\sqrt[3]{2z}$

$3\bullet3z^2\bullet\sqrt[3]{2z}$

$9z^2\ \sqrt[3]{2z}$

8) $\left(-2\sqrt{10}\right)\left(5\sqrt{30}\right)$

$-10\sqrt{2}\sqrt{5}\sqrt{5}\sqrt{6}$

$-10\sqrt{5}\sqrt{5}\sqrt{2}\sqrt{2}\sqrt{3}$

$-10\bullet5\bullet2\sqrt{3}$

$-100\sqrt{3}$

9) $\left(4\sqrt[3]{9mp^4}\right)\left(\sqrt[3]{3m^2p}\right)$

$4\sqrt[3]{27m^3p^5}$

$4\bullet3mp\bullet\sqrt[3]{p^2}$

$12mp\ \sqrt[3]{p^2}$

Examples: (FOIL)

10) $(2+\sqrt{5})(3+\sqrt{5})$

$6+2\sqrt{5}+3\sqrt{5}+\sqrt{25}$

$6+5\sqrt{5}+5$

$11+5\sqrt{5}$

11) $(7+\sqrt{3})(7-\sqrt{3})$

$49-\sqrt{9}$

$49-3$

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no need to do O & I from FOIL since this is a conjugate problem

12) $(1-2\sqrt{6})(2-\sqrt{2})$

$2-1\sqrt{2}-4\sqrt{6}\ -2\sqrt{12}$

13) $(9+\sqrt{7})^2=(9+\sqrt{7})(9+\sqrt{7})$

$81+9\sqrt{7}+9\sqrt{7}+\sqrt{49}$

$81+18\sqrt{7}+7$

$88+18\sqrt{7}$

ADD & SUBTRACT: Why can you add 2x + 3x and not 2x + 3y?

Can you add $5\sqrt{11} + 4\sqrt{13}$? How about $\sqrt[3]{32} - 6\sqrt[4]{18}$? **WHY?**

Examples: (Add/subtract)

14) $5\sqrt{14} + 7\sqrt{14} - \sqrt{14}$

don't think to hard:

$= 11\sqrt{14}$

15) $\sqrt[3]{4} + 6\sqrt{2} - 8\sqrt{2} + 7\sqrt[3]{4}$

combine like terms only:

$\sqrt[3]{4} + 7\sqrt[3]{4} + 6\sqrt{2} - 8\sqrt{2}$

$8\sqrt[3]{4} - 2\sqrt{2}$

Examples: (Simplify first)

16) $\sqrt{75} + 4\sqrt{12}$

factor trees first, then combine like terms:

$\sqrt{25}\sqrt{3} + 4\sqrt{4}\sqrt{3}$

$5\sqrt{3} + 4 \bullet 2\sqrt{3}$

$5\sqrt{3} + 8\sqrt{3}$

$13\sqrt{3}$

17) $\sqrt[3]{16} - \sqrt[3]{54}$

$\sqrt[3]{8} \bullet \sqrt[3]{2} - \sqrt[3]{27} \bullet \sqrt[3]{2}$

$2\sqrt[3]{2} - 3\sqrt[3]{2}$

$-\sqrt[3]{2}$

18) $\sqrt{98} - \sqrt{48} + \sqrt{27}$

$\sqrt{49}\sqrt{2} - \sqrt{16}\sqrt{3} + \sqrt{9}\sqrt{3}$

$7\sqrt{2} - 4\sqrt{3} + 3\sqrt{3}$

combine only like terms:

$7\sqrt{2} - \sqrt{3}$

19) $3\sqrt[3]{-125} + \sqrt[3]{-64}$

$\sqrt[3]{-125} = -5$

$\sqrt[3]{-64} = -4$

so… 3(-5)(-4)
= 60

20) $-4\sqrt{20} - 2\sqrt{500} + 3\sqrt[3]{16}$

$-4\sqrt{4}\sqrt{5} - 2\sqrt{100}\sqrt{5} + 3\sqrt[3]{8}\sqrt[3]{2}$

$-4 \bullet 2\sqrt{5} - 2 \bullet 10\sqrt{5} + 3 \bullet 2\sqrt[3]{2}$

$-8\sqrt{5} - 20\sqrt{5} + 6\sqrt[3]{2}$

$-28\sqrt{5} + 6\sqrt[3]{2}$

RATIONALIZING THE DENOMINATOR (not a whole lot of fun)

You may not have a root of any kind in the denominator of a fraction. Why? How can you divide something evenly by an irrational number (a decimal that goes on for ever)?

Answer: You can't! Therefore, the math gods say get rid of any radical (root) on the bottom of a fraction.

Examples: (no way they'll all be this easy)

1) $\sqrt[3]{\frac{7}{27}}$

$$\sqrt[3]{\frac{7}{27}} = \frac{\sqrt[3]{7}}{\sqrt[3]{27}} = \frac{\sqrt[3]{7}}{3}$$

2) $\sqrt{\frac{x^3}{y^6}}$

$$\sqrt{\frac{x^3}{y^6}} = \frac{\sqrt{x^3}}{\sqrt{y^6}} = \frac{x\sqrt{x}}{y^2}$$

called rationalizing the denominator

Examples: (square roots aren't so bad)

Remember how we got rid of the "*i*" from the denominator?

3) $\sqrt{\frac{5}{2}}$

$$\sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{\sqrt{4}} = \frac{\sqrt{10}}{2}$$

4) $\frac{3}{\sqrt{11}}$

$$\frac{3}{\sqrt{11}} \bullet \frac{\sqrt{11}}{\sqrt{11}} = \frac{3\sqrt{11}}{11}$$

5) $\frac{2}{\sqrt{c^5}}$

$$\frac{2}{\sqrt{c^5}} \bullet \frac{\sqrt{c}}{\sqrt{c}} = \frac{2}{\sqrt{c^6}} = \frac{2}{c^3}$$

Examples: (other roots aren't so good)

Need to use a 9 so the cubed root on the bottom will come out even

6) $\sqrt[3]{\frac{1}{3}}$

$$\sqrt[3]{\frac{1}{3}} = \frac{\sqrt[3]{1}}{\sqrt[3]{3}} \bullet \frac{\sqrt[3]{9}}{\sqrt[3]{9}} = \frac{\sqrt[3]{9}}{\sqrt[3]{27}} = \frac{\sqrt[3]{9}}{3}$$

7) $\frac{2}{\sqrt[3]{25x}}$

$$\frac{2}{\sqrt[3]{25x}} \bullet \frac{\sqrt[3]{5x^2}}{\sqrt[3]{5x^2}} = \frac{2\sqrt[3]{5x^2}}{\sqrt[3]{125x^3}} = \frac{2\sqrt[3]{5x^2}}{5x}$$

8) $\frac{1}{\sqrt[5]{8}}$

$$\frac{1}{\sqrt[5]{8}} \bullet \frac{\sqrt[5]{4}}{\sqrt[5]{4}} = \frac{1\sqrt[5]{4}}{\sqrt[5]{32}} = \frac{\sqrt[5]{4}}{2}$$

Examples: (conjugates)

Distribute on top. FOIL the bottom. Notice the "O"&"I" are unnecessary

9) $\frac{5}{3-\sqrt{2}}$

$$\frac{5}{3-\sqrt{2}} \bullet \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{15+5\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}-\sqrt{4}} = \frac{15+5\sqrt{2}}{9-2} = \frac{15+5\sqrt{2}}{7}$$

10) $\frac{2+\sqrt{3}}{4-\sqrt{3}}$

$$\frac{2+\sqrt{3}}{4-\sqrt{3}} \bullet \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{8+2\sqrt{3}+4\sqrt{3}+\sqrt{9}}{16-\sqrt{9}} = \frac{11+6\sqrt{3}}{13}$$

FOIL top, F & L only on bottom

11) $\frac{1+3\sqrt{5}}{2+\sqrt{6}}$

The $3+\sqrt{2}$ in #9 and the $4+\sqrt{3}$ in #10 are called conjugates. You need only change the sign in the middle.