Algebra II
Section 9-5

SOLVING < RATIONAL > INEQUALITIES

Steps for solving:

1) determine the restrictions.

2) solve the problem as if it were an equation.

3) create a number line with the solution & the restrictions.

4) test values from each section of the number line.

5) write a solution set based on the results from step 4.

Examples:

1) $\frac{1}{4a}+\frac{5}{8a}>\frac{1}{2}$

1) restriction: $a \neq 0$

2) $2\bullet\ \frac{1}{4a}+\frac{5}{8a}=\frac{1}{2}\ \bullet 4a$

$2\bullet\ \bullet 4a$

$\frac{2}{8a}+\frac{5}{8a}=\frac{4a}{8a}$

$2 + 5 = 4a$

$7 = 4\ a$

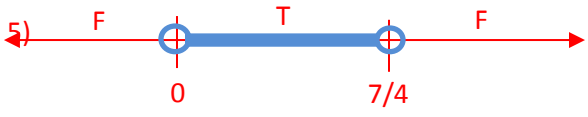
$7/4 = a$

3)

4) test a number to the left of zero, say (-1),
test a number between 0 & 7/4, say (+1),
test a number to the right of 7/4, say (+2), and
plug them into the original problem.

Test (-1): 1/-4 + 5/-8 > 1/2 ? False
Test (1): 1/4 + 5/8 > 1/2 ? True
Test (2): 1/8 + 5/16 > 1/2 ? False

5)



Solution set: { 0 < a < 7/4 }

2) $\frac{1}{3b}-\frac{2}{5b}<\frac{1}{15}$

1) restriction: $b \neq 0$

2) $5\bullet\ \frac{1}{3b}-\frac{2}{5b}\overset{\bullet 3}{=}\frac{1}{15}\ \bullet b$

$5\bullet\ \bullet 3\qquad\bullet b$

$\frac{5}{15b}-\frac{6}{15b}=\frac{b}{15b}$

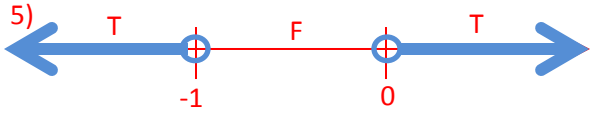
$5 - 6 = b$

$-1 = b$

3)

4) plug in (-2): 1/-6 – 2/-10 < 1/15 ? True
plug in (-0.5): 1/-1.5 – 2/-2.5 < 1/15 ? False
plug in (+1): 1/3 – 2/5 < 1/15 ? True

5)



Solution set: { b < -1 or b > 0 }

3) $1+\frac{5}{x-1}\le\frac{7}{6}$

1) restriction: $x \neq 1$

2) $6(x-1)\cdot 1+\frac{5}{x-1}=\frac{7}{6}\cdot(x-1)$ (•6 on the left, •(x-1) on the right)

$6(x-1)\cdot\frac{1}{1}+\frac{5}{x-1}=\frac{7}{6}\cdot(x-1)$ (•6 on the left, •(x-1) on the right)

$$\frac{6x-6}{6(x-1)}+\frac{30}{6(x-1)}=\frac{7x-7}{6(x-1)}$$

$6x - 6 + 30 = 7x - 7$

$6x + 24 = 7x - 7$

$24 = x - 7$

$31 = x$

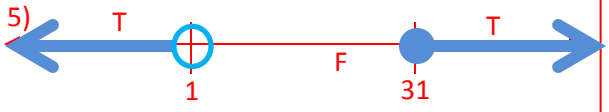
3)

4) plug in (0): $1 + 5/(0-1) \le 7/6$? True

plug in (2): $1 + 5/(2-1) \le 7/6$? ? False

plug in (32): $1 + 5/(32-1) \le 7/6$? True

5)



Solution set: $\{ x < 1 \text{ or } x \ge 31 \}$

4) $\frac{x}{x^2-9}+\frac{4}{x+3}\ge\frac{-5}{3-x}$

1) restrictions: $x \neq 3, -3$

2) factor first

$$\frac{x}{(x-3)(x+3)}+\frac{4}{x+3}=\frac{-5}{-1(x-3)}$$

2a) move the (-1) to the top

$$\frac{x}{(x-3)(x+3)}+\frac{4}{x+3}=\frac{(-1)(-5)}{(x-3)}$$

$\frac{x}{(x-3)(x+3)}+\frac{4}{x+3}\cdot\frac{(x-3)}{(x-3)}=\frac{5}{(x-3)}\cdot\frac{(x+3)}{(x+3)}$ (•(x-3) on the left, •(x+3) on the right)

$$\frac{x}{(x-3)(x+3)}+\frac{4x-12}{(x-3)(x+3)}=\frac{5x+15}{(x-3)(x+3)}$$

$x + 4x - 12 = 5x + 15$

$5x - 12 = 5x + 15$

$-12 = 15$

no solution, therefore the restrictions are the only values to test

3)

4) plug in -4: False plug in 0: True

plug in +4: False

Solutin set: $\{ -3 < x < 3 \}$

Cross multiply examples:

5) $\frac{n-4}{8}<\frac{n+1}{5}$

1) no restrictions

2) $5n - 20 < 8n + 8$

$-20 < 3n + 8$

$-28 < 3n$

$-28/3 < n$ or $n > -28/3$

Since there were no restrictions…

Solution set: $\{ n > -28/3 \}$

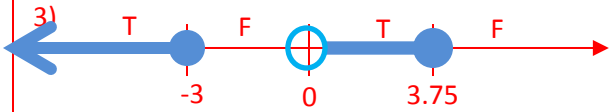
6) $\frac{m+15}{m}\ge\frac{4m}{3}$

1) restriction: $m \neq 0$

2) $3m + 45 \le 4m^2$

$0 = 4m^2 - 3m - 45$ run program: -3, 3.75

3)



4) plug in (-15): True plug in (-1): False

plug in (+1): True plug in (+5): False

Solution set: $\{ m \le -3 \text{ or } 0 < m \le 3.75 \}$