Chapter M Section 1 Introduction to matrices

Matrices are referred to by the number of rows by the number of columns.

2 X 2

$$\begin{bmatrix} -3 & 4 \\ -5 & -1 \end{bmatrix}$$

3 X 2

$$\begin{bmatrix} -3 & 4 \\ -5 & -1 \\ 10 & 6 \end{bmatrix}$$

2 X 3

$$\begin{bmatrix} -3 & 4 & 0 \\ -5 & -1 & 2 \end{bmatrix}$$

Solving for variables in a matrix.
Examples:

1) $$\begin{bmatrix} -3 & 8x \\ x+y & -1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -5 & -1 \end{bmatrix}$$

8x = 4　　　x + y = -5
x = 2 sub in -> 2 + y = -5
y = -7
x = 2, y = -7

2) $$\begin{bmatrix} 4x-3 & 3y \\ 7 & 13 \end{bmatrix} = \begin{bmatrix} 9 & -15 \\ 7 & 2z+1 \end{bmatrix}$$

4x – 3 = 9　　　3y = -15　　　13 = 2z + 1
4x = 12　　　y = -5　　　12 = 2z
x = 3　　　　　　6 = z
x = 3, y = -5, z = 6

3) $$\begin{bmatrix} x+3y \\ 3x+y \end{bmatrix} = \begin{bmatrix} -13 \\ 1 \end{bmatrix}$$

x + 3y = -13　　　3x + y = 1
Solve as a system of equations.
(substitution or elimination)

x = -3y – 13 sub in -> 3(-3y-13) + y = 1
-9y – 39 + y = 1
-8y – 39 = 1
-8y = 40
x = -3(-5) – 13　<-sub in　y = -5
x = 2
x = 2, y = -5

4) $$\begin{bmatrix} f+h & 7 & -3j \\ f-j & 0 & -5 \\ 6 & 1 & 2h \end{bmatrix} = \begin{bmatrix} 11 & 7 & 9 \\ k & 0 & -5 \\ 3f+j & 1 & 16 \end{bmatrix}$$

f + h = 11　　　-3j = 9　　　f – j = k　　　6 = 3f + j　　　2h = 16
j = -3　　　　　　h = 8
Substitute j = -3 and h = 8 to find the other values.
f + 8 = 11　　　3 – (-3) = k　　　Did not need
f = 3　　　6 = k　　　4th equation

f = 3, h = 8, j = -3, k = 6

Operations with matrices.
Addition/subtractions examples.

If A = $\begin{bmatrix} -3 & 4 \\ -5 & -1 \end{bmatrix}$ B = $\begin{bmatrix} 13 & 10 \\ -7 & 31 \end{bmatrix}$ C = $\begin{bmatrix} -6 & 9 & -2 \\ -5 & -3 & 8 \end{bmatrix}$

5) Find A + B

$$\begin{bmatrix} -3 + 13 & 4 + 10 \\ -5 - 7 & -1 + 31 \end{bmatrix} = \begin{bmatrix} 10 & 14 \\ -12 & 30 \end{bmatrix}$$

6) Find B – A

$$\begin{bmatrix} 13 - (-3) & 10 - 4 \\ -7 - (-5) & 31 - (-1) \end{bmatrix} = \begin{bmatrix} 16 & 6 \\ -2 & 32 \end{bmatrix}$$

7) Find A + C

Matrices much match in dimension to perform addition or subtraction, therefore A + C is not possible.

Multiplying by a scalar examples.

8) Find 3A

$$\begin{bmatrix} 3(-3) & 3(4) \\ 3(-5) & 3(-1) \end{bmatrix} = \begin{bmatrix} -9 & 12 \\ -15 & -3 \end{bmatrix}$$

9) Find -4C

$$\begin{bmatrix} -4(-6) & -4(9) & -4(-2) \\ -4(-5) & -4(-3) & -4(8) \end{bmatrix} = \begin{bmatrix} 24 & -36 & -8 \\ 20 & 12 & -32 \end{bmatrix}$$

Multiplying matrices. If A = $\begin{bmatrix} -3 & 4 \\ -5 & -1 \end{bmatrix}$ B = $\begin{bmatrix} 13 & 10 \\ -7 & 31 \end{bmatrix}$ C = $\begin{bmatrix} -6 & 9 & -2 \\ -5 & -3 & 8 \end{bmatrix}$ D = $\begin{bmatrix} 6 & -6 \\ -1 & -2 \\ 0 & 7 \end{bmatrix}$

10) Find AB

11) Find BC

12) Find BD

AB =

$$\begin{bmatrix} -3(13) + 4(-7) & -3(10) + 4(31) \\ -5(13) + -1(-7) & -5(10) + -1(31) \end{bmatrix} = \begin{bmatrix} -67 & 94 \\ -58 & -81 \end{bmatrix}$$

BD is impossible.
The number of columns in the first matrix must be the same as the number of rows in the second one.

BC =

$$\begin{bmatrix} -13(-6) + 10(-5) & 13(9) + 10(-3) & 13(-2) + 10(8) \\ -7(-6) + 31(-5) & -7(9) + 31(-3) & -7(-2) + 31(8) \end{bmatrix} = \begin{bmatrix} 28 & 87 & 54 \\ -113 & -156 & 262 \end{bmatrix}$$

Applications.

School	First place	Second place	Third place
Central	4	7	3
Franklin	8	9	1
Hayes	10	5	3
Lincoln	3	3	6

The results from a quad swim meet for four schools is shown in the chart. If 7 points was awarded for each first place finish, 4 for second and 2 for third, in what order did the four schools finish?

$$\begin{bmatrix} 4 & 7 & 3 \\ 8 & 9 & 1 \\ 10 & 5 & 3 \\ 3 & 3 & 6 \end{bmatrix} \bullet \begin{bmatrix} 7 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4(7) + 7(4) + 3(2) \\ 8(7) + 9(4) + 1(2) \\ 10(7) + 5(4) + 3(2) \\ 3(7) + 3(4) + 6(2) \end{bmatrix} = \begin{bmatrix} 62 \\ 94 \\ 96 \\ 45 \end{bmatrix}$$

1^{st} place: Hayes
2^{nd} place: Franklin
3^{rd} place: Central
4^{th} place: Lincoln