

Chapter M Section 2 Determinants

2 X 2 Determinants.

Talk about easy, just multiply each set of diagonals, then subtract the two answers.

$$1) \begin{vmatrix} -2 & 5 \\ 6 & 8 \end{vmatrix}$$

$$\begin{aligned} &(-2)(8) - (5)(6) \\ &= -16 - 30 \\ &= -46 \\ &\text{Determinant is } -46. \end{aligned}$$

$$2) \begin{vmatrix} 18 & -11 \\ -5 & 22 \end{vmatrix}$$

$$\begin{aligned} &(18)(22) - (-11)(-5) \\ &= 396 - 55 \\ &= 341 \\ &\text{Determinant is } 341. \end{aligned}$$

3 X 3 (or greater) Determinants.

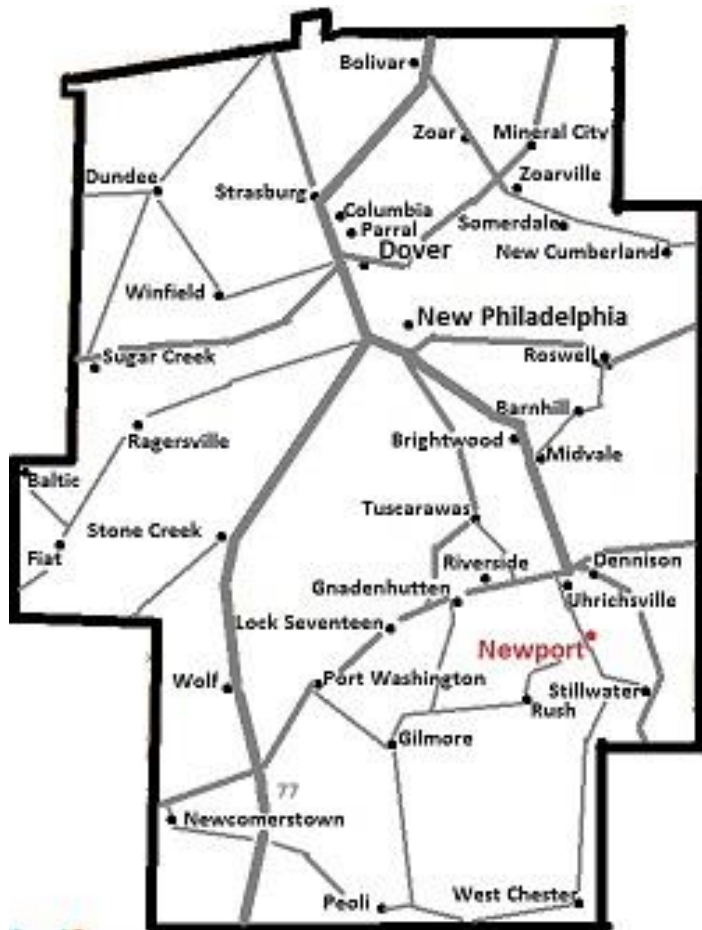
Talk about not easy at all. Combine ALL the right facing diagonals then subtract ALL the left facing diagonals.

$$3) \begin{vmatrix} 3 & 1 & -2 \\ -4 & 2 & 5 \\ 2 & 7 & -1 \end{vmatrix} \quad \begin{matrix} 3 & 1 \\ -4 & 2 \\ 2 & 7 \end{matrix}$$

$$\begin{aligned} &\text{Right diagonals:} \\ &3(2)(-1) + 1(5)(2) + -2(-4)(7) = 72 \\ &\text{Left diagonals:} \\ &(-2)(2)(2) + (3)(5)(7) + (1)(-4)(-1) = 101 \\ &\text{Subtract the two: } 72 - 101 \\ &\text{Determinant} = -29 \end{aligned}$$

$$4) \begin{vmatrix} 10 & 6 & 5 \\ 3 & -5 & 11 \\ 1 & 0 & -9 \end{vmatrix} \quad \begin{matrix} 10 & 6 \\ 3 & -5 \\ 1 & 0 \end{matrix}$$

$$\begin{aligned} &\text{Right diagonals:} \\ &(10)(-5)(-9) + (6)(11)(1) + (5)(3)(0) = 516 \\ &\text{Left diagonals:} \\ &(5)(-5)(1) + (10)(11)(0) + (6)(3)(-9) = -187 \\ &\text{Subtract the two: } 516 - (-187) \\ &\text{Determinant} = 703 \end{aligned}$$



Applications.

As King of Central Tuscarawas County, you have inherited all the land between Port Washington, Dennison and Roswell. If one were to lay the image above onto a coordinate grid, Port would be at (0, 0), Dennison at (12, 3) and Roswell at (13, 9). If each "grid" unit is 1 mile in length, how many square miles of land do you own?

$$A = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 12 & 3 & 1 \\ 13 & 9 & 1 \end{vmatrix} \begin{vmatrix} 0 & 0 \\ 12 & 3 \\ 13 & 9 \end{vmatrix}$$

Right diagonals:

$$(0)(3)(1) + (0)(1)(13) + (1)(12)(9) = 108$$

Left diagonals:

$$(1)(3)(13) + (0)(1)(9) + (0)(12)(1) = 39$$

$$\text{Subtract the two: } 108 - 39 = 69$$

$$\text{Determinant} = 69$$

$$\text{Area} = \frac{1}{2} (69) = 34.5 \text{ square miles.}$$