

Determine if the following are polynomial functions (yes or no). Then find the zeros (roots), if they exist, for all the functions given

	YES/NO	ZEROS (ROOTS)
1) $f(x) = x^3 - 2x^2 - 48x$	yes	-6, 0, 8
2) $g(x) = \frac{x^2 - 1}{3}$	yes	$\pm 1$
3) $h(x) = 3x^{-2} + 1$	no	$\pm i\sqrt{3}$
4) $k(x) = \frac{2}{3}x - 14$	yes	21
5) $f(x) = \frac{x^2 + 7x + 6}{x^2 - 1}$	no	-6
6) $g(x) = -11$	yes	none

$$\begin{aligned}
 1) \quad & x^3 - 2x^2 - 48x = 0 \\
 & x(x^2 - 2x - 48) = 0 \\
 & x(x - 8)(x + 6) = 0 \\
 & x = 0, x = 8, x = -6
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & (3) \frac{x^2 - 1}{3} = 0(3) \\
 & x^2 - 1 = 0 \\
 & (x - 1)(x + 1) = 0 \\
 & x = 1, x = -1
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & 3x^{-2} + 1 = 0 \\
 & \frac{3}{x^2} + 1 = 0 \\
 & 3 + x^2 = 0 \\
 & x^2 = -3 \\
 & x = \pm i\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \frac{2}{3}x - 14 = 0 \quad \left(\frac{3}{2}\right) \frac{2}{3}x = 14 \left(\frac{3}{2}\right) \\
 & \frac{2}{3}x = 14 \quad \quad \quad x = 21
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & (x^2 - 1) \frac{x^2 + 7x + 6}{x^2 - 1} = 0(x^2 - 1) \\
 & x^2 + 7x + 6 = 0 \\
 & (x + 1)(x + 6) = 0 \\
 & x = -1, x = -6, \text{ but } x \neq -1
 \end{aligned}$$

$$\begin{aligned}
 6) \quad & -11 = 0 \\
 & \text{False statement,} \\
 & \text{therefore no} \\
 & \text{zeros.}
 \end{aligned}$$

Use the functions  $f(x) = 3x^2 - x + 15$  to find the following values.

7)  $f(-6) =$  \_\_\_\_\_

$$\begin{aligned}
 f(-6) &= 3(-6)^2 - (-6) + 15 \\
 &= 3(36) + 6 + 15 \\
 &= 108 + 6 + 15 \\
 &= 129
 \end{aligned}$$

8)  $f(-2i) =$  \_\_\_\_\_

$$\begin{aligned}
 f(-2i) &= 3(-2i)^2 - (-2i) + 15 \\
 &= 3(4i^2) + 2i + 15 \\
 &= 3(-4) + 2i + 15 \\
 &= -12 + 2i + 15 = 3 + 2i
 \end{aligned}$$

Use the functions  $g(x) = x^2(x - 5) - 2$  to find the following values.

9)  $g(-2\sqrt{3}) =$  \_\_\_\_\_

$$\begin{aligned}
 g(-2\sqrt{3}) &= (-2\sqrt{3})^2(-2\sqrt{3} - 5) - 2 \\
 &= (4\sqrt{9})(-2\sqrt{3} - 5) - 2 \\
 &= 12(-2\sqrt{3} - 5) - 2 \\
 &= -24\sqrt{3} - 60 - 2 \\
 &= -24\sqrt{3} - 62
 \end{aligned}$$

$$\begin{aligned}
 10) \quad g(2i + 1) &= (2i + 1)^2(2i + 1 - 5) - 2 \\
 &= (2i + 1)(2i + 1)(2i - 4) - 2 \\
 &= (4i^2 + 2i + 2i + 1)(2i - 4) - 2 \\
 &= (-4 + 4i + 1)(2i - 4) - 2 \\
 &= (-3 + 4i)(2i - 4) - 2 \\
 &= -6i + 12 + 8i^2 - 16i - 2 \\
 &= 10 - 22i - 8 \\
 &= 2 - 22i
 \end{aligned}$$

Use synthetic substitution to find the following values.

$$f(x) = x^3 - 3x^2 - 8x + 21$$

$$g(x) = 4x^4 + x^2 - 7x - 11$$

11)  $f(-2) = \underline{\quad 17 \quad}$

13)  $g(-1) = \underline{\quad 1 \quad}$

12)  $f(5) = \underline{\quad 31 \quad}$

14)  $g(1/2) = \underline{\quad -14 \quad}$

$$\begin{array}{r|rrrr} -2 & 1 & -3 & -8 & 21 \\ & & -2 & 10 & -4 \\ \hline & 1 & -5 & 2 & 17 \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -3 & -8 & 21 \\ & & 5 & 10 & 10 \\ \hline & 1 & 2 & 2 & 31 \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 4 & 0 & 1 & -7 & -11 \\ & & -4 & 4 & -5 & 12 \\ \hline & 4 & -4 & 5 & -12 & 1 \end{array}$$

$$\begin{array}{r|rrrrr} 1/2 & 4 & 0 & 1 & -7 & -11 \\ & & 2 & 1 & 1 & -3 \\ \hline & 4 & 2 & 2 & -6 & -14 \end{array}$$

Divide the polynomials using whatever method seems appropriate.

15)  $(3x^3 + 4x^2 - 76x + 50) \div (x - 4)$

16)  $(2x^4 - 7x^2 - 14) \div (2x^2 + 1) = x^2 - 4 - \frac{10}{2x^2 + 1}$

$$\begin{array}{r|rrrr} 4 & 3 & 4 & -76 & 50 \\ & & 12 & 64 & -48 \\ \hline & 3 & 16 & -12 & 2 \\ & & & & \frac{2}{x-4} \end{array}$$

$= 3x^2 + 16x - 12 + \frac{2}{x-4}$

$$\begin{array}{r} x^2 + 0x - 4 \\ 2x^2 + 0x + 1 \overline{) 2x^4 + 0x^3 - 7x^2 + 0x - 14} \\ \underline{-2x^4 + 0x^3 - x^2} \phantom{-14} \\ -8x^2 + 0x - 14 \\ \underline{-18x^2 - 0x + 4} \\ -10 \end{array}$$

Determine if the second polynomial is a factor of the first one given (yes or no). Must show proof!

17)  $x^3 + x^2 - 5x - 9; x - 2$

18)  $x^4 - 5x^3 + 30x - 36; x - 3$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -5 & -9 \\ & & 2 & 6 & 2 \\ \hline & 1 & 3 & 1 & (-7) \end{array} \quad \text{No!}$$

$$\begin{array}{r|rrrrr} 3 & 1 & -5 & 0 & 30 & -36 \\ & & 3 & -6 & -18 & 36 \\ \hline & 1 & -2 & -6 & 12 & (0) \end{array} \quad \text{Yes!}$$

Given a polynomial and one or more of its zeros, find the remaining zeros (roots).

19)  $x^3 + 10x^2 + 26x + 8; \text{ zero} = -4$

20)  $2x^5 + x^4 - 4x^2 - 32x; \text{ roots} = \pm 2$   
 $= x(2x^4 + x^2 - 4x - 32); \text{ root} = 0$

$$\begin{array}{r|rrrr} -4 & 1 & 10 & 26 & 8 \\ & & -4 & -24 & -8 \\ \hline & 1 & 6 & 2 & 0 \end{array}$$

(run program or use quadratic formula)  
 zeros =  $\underline{-3 \pm \sqrt{7}}$

$$\begin{array}{r|rrrrr} 2 & 2 & 1 & 0 & -4 & -32 \\ & & 4 & 10 & 20 & 32 \\ \hline & 2 & 5 & 10 & 16 & 0 \\ & & -4 & -2 & -16 & \\ \hline & 2 & 1 & 8 & 0 \end{array}$$

(program or quad form) =  $\underline{\underline{\frac{-1 \pm 3i\sqrt{7}}{4}}}$