COLLEGE REVIEW MATH
REVIEW SECT. 2F & 2G

Name KEY

Solve each equation by "grouping".

1) $2x^3 + 7x^2 - 18x - 63 = 0$

$2x^3 - 18x + 7x^2 - 63 = 0$
$2x(x^2 - 9) + 7(x^2 - 9) = 0$
$(2x + 7)(x^2 - 9) = 0$
$(2x + 7)(x - 3)(x + 3) = 0$
$x = -7/2, x = \pm 3$

2) $3x^5 - 1 + x^3 - 3x^2 = 0$

$3x^5 + x^3 - 3x^2 - 1 = 0$
$x^3(3x^2 + 1) - 1(3x^2 + 1) = 0$
$(x^3 - 1)(3x^2 + 1) = 0$
$(x - 1)(x^2 + x + 1)(3x^2 + 1) = 0$
$x = 1$, use quad formula or program
$x = 1, x = -0.5 \pm 0.9i, x = \pm 0.6i$

Solve each equation using quadratic "substitution".

3) $2x^4 - 7x^2 - 4 = 0$

$2y^2 - 7y - 4 = 0$
$(2y + 1)(y - 4) = 0$
$y = -1/2, \quad y = 4$

$x^2 = -1/2$ $\quad\quad$ $x^2 = 4$
$\sqrt{x^2} = \sqrt{-1/2}$ $\quad\quad$ $\sqrt{x^2} = \sqrt{4}$
$x = \pm 0.7i$ $\quad\quad$ **$x = \pm 2$**

4) $x - x^{1/2} = 30$

$y^2 - y - 30 = 0$
$(y - 6)(y + 5) = 0$
$y = 6, \quad y = -5$

$x^{1/2} = 6$ $\quad\quad$ $x^{1/2} = -5$
$\sqrt{x} = 6$ $\quad\quad$ $\sqrt{x} = -5$
$x = 36$ $\quad\quad$ **Ø**

List all possible p's, q's and p/q's for the following polynomial equations. Do not solve.

5) $3x^4 + 6x^2 - x - 6 = 0$

p: ± 1, 2, 3, 6
q: ± 1, 3
p/q: ± 1, 2, 3, 6, 1/3, 2/3

6) $4x^3 + 9x^2 = 10$

p: ± 1, 2, 5, 10
q: ± 1, 2, 4
p/q: ± 1, 2, 5, 10, 1/2, 1/4, 5/2, 5/4

Solve each equation using the rational root theorem (mind your p's & q's).

7) $3x^3 - 4x^2 - 35x + 12 = 0$

p/q: ± 1, 2, 3, 4, 6, 12, 1/3, 2/3, 4/3

4	3	-4	-35	12
		12	32	-12
	3	8	-3	0

$3x^2 + 8x - 3 = 0$
$(3x - 1)(x + 3) = 0$
$x = 4, x = 1/3, x = -3$

8) $2x^4 + 18x^3 + 32x^2 - 36x = 16$

$2(x^4 + 9x^3 + 16x^2 - 18x - 8) = 0$ p/q: ± 1, 2, 4, 8

1	1	9	16	-18	-8
		1	10	26	8
	1	10	26	8	0
-4		-4	-24	-8	
	1	6	2	0	

quadratic formula or program
$x = 1, x = -4, x = -0.4, x = -5.6$

9) $2x^5 + 14x^4 + 28x^3 + 56x^2 + 80x = 0$

$2x(x^4 + 7x^3 + 14x^2 + 28x + 40) = 0$ p/q: ± 1, 2, 4, 5, 8, 10, 20, 40

-2	1	7	14	28	40
		-2	-10	-8	-40
	1	5	4	20	0

-5	1	5	4	20
		-5	0	-20
	1	0	4	0

$x^2 + 4 = 0$
$x^2 = -4$
$x = \pm 2i$ **$x = 0, -2, -5, \pm 2i$**

Find the sum and product of the roots for each equation.

10) $8y^2 + 6y - 3 = 0$

sum = $-b/a = -6/8 = -3/4$

prod = $c/a = -3/8$

11) $x^3 + x^2 - x + 12 = 0$

sum = $-b/a = -1/1 = -1$

prod = $-c/a = -12/1 = -12$

12) $10x^6 - 3x^4 + x - 8 = 0$

sum = $-b/a = 0/10 = 0$

prod = $c/a = -8/10 = -4/5$

13) $9n^5 + 6n^4 + n^3 = 24$

sum = $-b/a = -6/9 = -2/3$

prod = $-c/a = -(-24)/9 = 8/3$

Use the sum and product formulas to write a quadratic equation for the given roots.

14) 1/2 and 3/5

sum: $1/2 + 3/5 = 11/10$
prod: $(1/2)(3/5) = 3/10$
$a = 10, b = -11, c = 3$

$10x^2 - 11x + 3 = 0$

15) $-4 \pm 3i$

sum: $-4 + 3i + -4 - 3i = -8$ or $-8/1$
prod: $(-4 + 3i)(-4 - 3i) = 16 - 9i^2$
$= 16 + 9 = 25$ or $25/1$
$a = 1, b = 8, c = 25$

$x^2 + 8x + 25 = 0$

16) $4 \pm \sqrt{7}$

sum: $4 + \sqrt{7} + 4 - \sqrt{7} = 8$ or $8/1$
prod: $(4 + \sqrt{7})(4 - \sqrt{7}) = 16 - 7 = 9$ or $9/1$
$a = 1, b = -8, c = 9$

$x^2 - 8x + 9 = 0$

17) $\dfrac{6 \pm 2i\sqrt{3}}{5}$

sum: $\dfrac{6 + 2i\sqrt{3}}{5} + \dfrac{6 - 2i\sqrt{3}}{5} = \dfrac{12}{5} = \dfrac{60}{25}$

prod: $\dfrac{6 + 2i\sqrt{3}}{5} \bullet \dfrac{6 - 2i\sqrt{3}}{5} = \dfrac{36 - 4i^2\sqrt{9}}{25} = \dfrac{36 + 12}{25} = \dfrac{48}{25}$

$a = 25, b = -60, c = 48$

$25x^2 - 60x + 48 = 0$

Write the cubic equation that has the given roots.

18) $7 \pm \sqrt{5}$ and -2

sum: $7 + \sqrt{5} + 7 - \sqrt{5} = 14$ or $14/1$
prod: $(7 + 5)(7 - \sqrt{5}) = 49 - 5 = 44$ or $44/1$
$a = 1, b = -14, c = 44$

$= (x^2 - 14x + 44)(x + 2)$

x	x^3	$-14x^2$	$44x$
2	$2x^2$	$-28x$	88

$x^3 - 12x^2 + 16x + 88 = 0$

19) $\dfrac{1 \pm i\sqrt{3}}{2}$ and 3

sum: $\dfrac{1 + i\sqrt{3}}{2} + \dfrac{1 - i\sqrt{3}}{2} = \dfrac{2}{2} = \dfrac{1}{1}$

prod: $\dfrac{1 + i\sqrt{3}}{2} \bullet \dfrac{1 - i\sqrt{3}}{2} = \dfrac{1 - i^2\sqrt{9}}{4} = \dfrac{1 + 3}{4} = \dfrac{1}{1}$

$= (x^2 - x + 1)(x - 3)$

x	x^3	$-x^2$	x
-3	$-3x^2$	$3x$	-3

$x^3 - 4x^2 + 4x - 3 = 0$