

The i -chart:

$$\begin{aligned}\sqrt{-1} &= i \\ i^2 &= -1 \\ i^3 &= -i \\ i^4 &= 1\end{aligned}$$



ADDITION/SUBTRACTION examples:

1) $\sqrt{-121} + \sqrt{-64} - \sqrt{-4}$

$$\begin{aligned}&= 11i + 8i - 2i \\ &= 17i\end{aligned}$$

2) $3\sqrt{-50} + 2\sqrt{-18}$

$$\begin{aligned}&= 3i\sqrt{25}\sqrt{2} + 2i\sqrt{9}\sqrt{2} \\ &= 3i \cdot 5\sqrt{2} + 2i \cdot 3\sqrt{2} \\ &= 15i\sqrt{2} + 6i\sqrt{2} \\ &= 21i\sqrt{2}\end{aligned}$$

3) $(10 - 7i) + (-4 + 12i)$

$$\begin{aligned}&\text{just combine like terms} \\ &= 6 + 5i\end{aligned}$$

4) $(3 + 2i) - (5 - 6i)$

$$\begin{aligned}&= 3 + 2i - 5 + 6i \\ &= -2 + 8i\end{aligned}$$

DISTRIBUTIVE PROPERTY examples:

5) $8(-1 - 2i) + 5(2 + 3i)$

$$\begin{aligned}&= -8 - 16i + 10 + 15i \\ &= 2 - i\end{aligned}$$

6) $\frac{1}{2}(22 + 12i) - 2(4i - 6)$

$$\begin{aligned}&= 11 + 6i - 8i + 12 \\ &= 23 - 2i\end{aligned}$$

MULTIPLICATION examples:

7) $\sqrt{-16}(5\sqrt{-1})$

$$\begin{aligned}&= 4i(5i) \\ &= 20i^2 = 20(-1) = -20\end{aligned}$$

8) $\sqrt{-3} \cdot \sqrt{-5} \cdot \sqrt{-7}$

$$\begin{aligned}&= i\sqrt{3} \cdot i\sqrt{5} \cdot i\sqrt{7} \\ &= i^3\sqrt{105} \\ &= -i\sqrt{105}\end{aligned}$$

9) $(-4\sqrt{6})(\sqrt{-15})$

$$\begin{aligned}&= -4\sqrt{2}\sqrt{3} \cdot i\sqrt{3}\sqrt{5} \\ &= -4 \cdot 3i\sqrt{2}\sqrt{5} \\ &= -12i\sqrt{10}\end{aligned}$$

DIVISION examples:

10) $\frac{\sqrt{-125}}{\sqrt{-5}}$

$$\begin{aligned}&= \frac{i\sqrt{5}\sqrt{25}}{i\sqrt{5}} \\ &= \sqrt{25} = 5\end{aligned}$$

11) $\sqrt{-\frac{20}{49}}$

$$\begin{aligned}&= i\frac{\sqrt{20}}{\sqrt{49}} = i\frac{\sqrt{4}\sqrt{5}}{7} \\ &= \frac{2i\sqrt{5}}{7}\end{aligned}$$

JUST "i" examples:

12) i^{131}

$$\text{Divide 131 by 4, remainder = 3, therefore } i^{131} = i^3 = -i$$

13) $i^{17} + i^{43}$

$$\begin{aligned}i^{17} &= i^1, i^{43} = i^3 \text{ so,} \\ &= i + (-i) = 0\end{aligned}$$

14) i^{-2}

$$= \frac{1}{i^2} = \frac{1}{-1} = -1$$

FOIL examples:

15) $(2 - 4i)(3 + 2i)$

$$\begin{aligned} &= 6 + 4i - 12i - 8i^2 \\ &= 6 - 8i - 8(-1) \\ &= 6 - 8i + 8 \\ &= 14 - 8i \end{aligned}$$

16) $(1 + \sqrt{-3})(5 + 2\sqrt{-3})$

$$\begin{aligned} &= (1 + i\sqrt{3})(5 + 2i\sqrt{3}) \\ &= 5 + 2i\sqrt{3} + 5i\sqrt{3} + 2i^2\sqrt{9} \\ &= 5 + 7i\sqrt{3} + 2(-1)(3) \\ &= 5 + 7i\sqrt{3} - 6 \\ &= -1 + 7i\sqrt{3} \end{aligned}$$

17) $(-3 + 5i)^2$

$$\begin{aligned} &= (-3 + 5i)(-3 + 5i) \\ &= 9 - 15i - 15i + 25i^2 \\ &= 9 - 30i - 25 \\ &= -16 - 30i \end{aligned}$$

RATIONALIZE THE DENOMINATOR examples:

18) $\frac{\sqrt{-2}}{\sqrt{-10}}$

$$= \frac{i\sqrt{2}}{i\sqrt{10}} = \frac{\sqrt{1}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{25}} = \frac{\sqrt{5}}{5}$$

rationalize the denominator

19) $\frac{4}{\sqrt{-20}}$

$$\begin{aligned} &= \frac{4}{i\sqrt{4}\sqrt{5}} = \frac{4}{2i\sqrt{5}} = \frac{2}{i\sqrt{5}} \cdot \frac{i\sqrt{5}}{i\sqrt{5}} \\ &= \frac{2i\sqrt{5}}{i^2\sqrt{25}} = \frac{2i\sqrt{5}}{-1(5)} = -\frac{2i\sqrt{5}}{5} \end{aligned}$$

20) i^{-7}

$$= \frac{1}{i^7} = \frac{1}{i^3} \cdot \frac{i}{i} = \frac{i}{i^4} = \frac{i}{1} = i$$

CONJUGATE examples:

distribute top
FOIL bottom, but you may skip the O&I since they will cancel

FOIL the top
F&L on bottom

21) $\frac{3}{6 - 4i}$

$$\begin{aligned} &\cdot \frac{6 + 4i}{6 + 4i} \\ &= \frac{18 + 12i}{36 + 24i - 24i - 16i^2} \\ &= \frac{18 + 12i}{36 + 16} \\ &= \frac{18 + 12i}{52} \\ &= \frac{9 + 6i}{26} \end{aligned}$$

sometimes it is necessary to reduce, but it must be all three parts or not at all

22) $\frac{10 + i}{10 - i}$

$$\begin{aligned} &\cdot \frac{10 + i}{10 + i} \\ &= \frac{100 + 10i + 10i + i^2}{100 - i^2} \\ &= \frac{100 + 20i - 1}{100 + 1} \\ &= \frac{99 + 20i}{101} \end{aligned}$$