College Review Math
Section 2B

SYNTHETIC DIVISION
REMAINDER & FACTOR THEOREMS

The Remainder Theorem

When a polynomial P(x) is divided by (x – a), the remainder is P(a)

Example: $f(x) = 2x^3 - 5x^2 + x - 7$; $x - 3$

3				
	2	-5	1	-7
		6	3	12
	2	1	4	5

The answer? $2x^2 + x + 4 + \frac{5}{x-3}$

also…
f(3) = 5

Your turn:

1) $-3x^4 + 2x^3 + 11x^2 - 3$; $x + 2$

-2					
	-3	2	11	1	-3
		6	-16	10	-22
	-3	8	-5	11	-25

$= -3x^3 + 8x^2 - 5x + 11 - \frac{25}{x+2}$

and…
f(-2) = -25

2) $16x^2 - 8x - 18$; $2x - 1$

-1/2			
	8	-4	-9
		-4	4
	8	-8	-5

Must divide everything by "2" first in order to use syn. div.

$= 8x - 8 - \frac{5}{x - 1/2}$

$= 8x - 8 - \frac{10}{2x - 1}$

and…
f(-1/2) = -5

The Factor Theorem

For a polynomial P(x), x – a is a factor if and only if P(a) = 0

Determine if the 2nd polynomial is a factor of the first one.

Example: $x^3 + x^2 - 10x + 8$; $x + 4$

-4				
	1	1	-10	8
		-4	12	-8
	1	-3	2	0

How can you tell if it is or isn't a factor? It is because the remainder was "zero".

The Factor Theorem

Finding the remaining roots for a polynomial equation.
(given one or more roots to start)

Example: $x^3 + x^2 - 10x + 8 = 0$; root = -4

-4	1	1	-10	8
		-4	12	-8
	1	-3	2	0

How can you determine the two remaining roots? Reinsert the variable, then factor.

$1x^2 - 3x + 2 = 0$
$(x - 2)(x - 1) = 0$
$x = 2$ and $x = 1$

Your turn:

3) $3x^3 + 2x^2 - 27x - 18 = 0$
root; $x = 3$

3	3	2	-27	-18
		9	33	18
	3	11	6	0

$3x^2 + 11x + 6 = 0$
$(3x + 2)(x + 3) = 0$
$x = -2/3$ and $x = -3$

4) $x^4 + 2x^3 - 10x^2 + 10x - 75 = 0$
roots; $x = -5$, $x = 3$

	1	2	-10	10	-75
-5		-5	15	-25	75
	1	-3	5	-15	0
3		3	0	15	
	1	0	5	0	

Since $1x^2 + 5$ does not factor, solve another way…
$x^2 + 5 = 0$
$x^2 = -5$ square root both sides
$x = \pm i\sqrt{5}$

5) $8x^4 + 32x^3 - x - 4 = 0$ root; $x = -4$

-4	8	32	0	-1	-4
		-32	0	0	4
	8	0	0	-1	0

$8x^3 - 1 = 0$
$(2x - 1)(4x^2 + 2x + 1) = 0$
$x = 1/2$, use quadratic formula
$x = .3 \pm .4i$ or $\frac{-1 \pm i\sqrt{3}}{4}$