

# GRAPHING POLYNOMIAL FUNCTIONS

Find the roots for each polynomial function.

1)  $f(x) = x(x - 3)(x + 4)$

$x = \underline{0, 3, -4}$

2)  $y = x^2(x - 7)(x + 6)$

$x = \underline{0, 7, -6}$

\*\*Now consider the graphs for each using your graphing calculator. What interesting coincidence occurs for each? For #2, change the WINDOW settings so that y-min = -100 and y-max = 100

The graph intersects the x-axis at the solutions.

\*\*How was the shape of the graph different in #2 compared to #1?

It touched, but did not cross the x-axis at  $x = 0$ .  
(did a U-turn)

3)  $y = (x + 5)(x - 1)^2$

$x = \underline{-5, 1}$

4)  $y = x^3(x + 3)(x - 5)^2$

$x = \underline{0, -3, 5}$

\*\*You may want to reset the WINDOW for #4 to -1000 & 1000 for the y, and -6 & 6 for the x

\*\*How did the graph for #4 differ?

The graph for #4 snaked (or twisted) through the x-axis at  $x = 0$

TYPES OR ROOTS:      SINGLES, DOUBLES OR TRIPLES

A single root (or exponent of 1) passes **directly thru** the x-axis at the root.

A double root (or exponent of 2) **changes direction** at the x-axis (or it forms a "U" at the root.

A triple root (or exponent of 3) **twists** its way thru the x-axis at the root.

Determine how many of each type of root (single, double or triple) each function has.

1)  $f(x) = (x - 10)(x + 1)^2(x - 2)$

single: 2

double: 1

triple: 0

2)  $f(x) = -3x^2(x + 4)^3(x + 9)(x + 7)^2$

single: 1

double: 2

triple: 1

## SIGN ANALYSIS

Determine if the graph for each function is above or below the x-axis for each value given. Simply plug the number in to the equation, keeping track of the sign for each portion.

EXAMPLE:  $f(x) = x^3(x-7)(x+2)^2$  @  $x = -1$

1)  $f(x) = (x+11)^2(x-4)(x+4)$  @  $x = -3$

plug in each part&

keep track of sign:  $x^3$ :  $(-1)^3 =$  negative  
 $(x-7)$   $(-1-7) =$  negative  
 $(x+2)^2$   $(-1+2)^2 =$  positive  
 answer = negative X negative X positive = positive

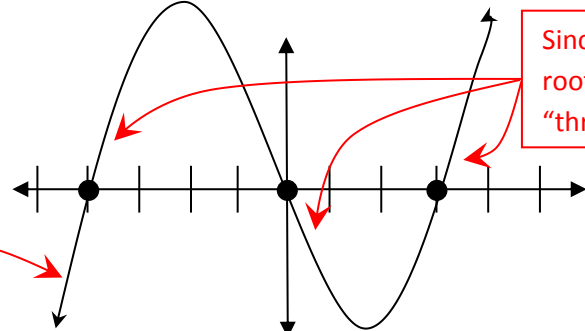
Therefore,  $x@-1$  is above the x-axis

$(-3+11)^2$ , short cut; anything<sup>2</sup> = positive  
 $(-3-4) =$  negative  
 $(-3+4) =$  positive  
 answer = positive X negative X positive = negative

Therefore,  $x@-3$  is below the x-axis

## GRAPH USING SIGN ANALYSIS & TYPE OF ROOT

EXAMPLE: Sketch the graph for  $y = x(x+4)(x-3)$



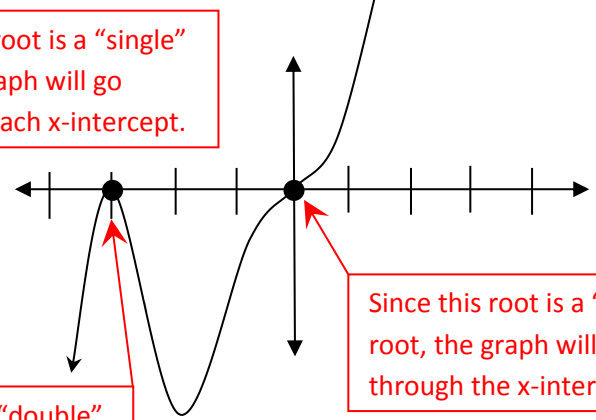
Since each root is a "single" root, the graph will go "through" each x-intercept.

Sign analysis for  $x = -5$ , yields a negative answer, so it must be below the x-axis

Your turn. Sketch the following.

1)  $f(x) = (x+6)^3(x-1)^2(x-5)$

EXAMPLE:  $y = x^3(x+3)^2$

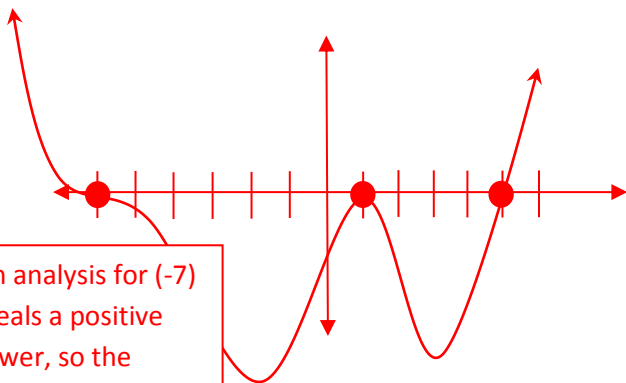


Since this root is a "triple" root, the graph will "twist" through the x-intercept.

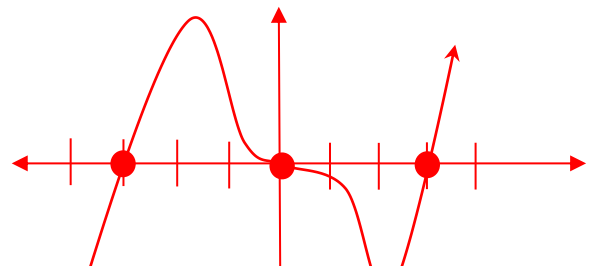
Since this root is a "double" root, the graph will do a "U-turn" at the x-intercept.

2)  $y = 2x^5 - 18x^3$

$y = 2x^5 - 18x^3$   
 $y = 2x^3(x^2 - 9)$   
 $y = 2x^3(x-3)(x+3)$

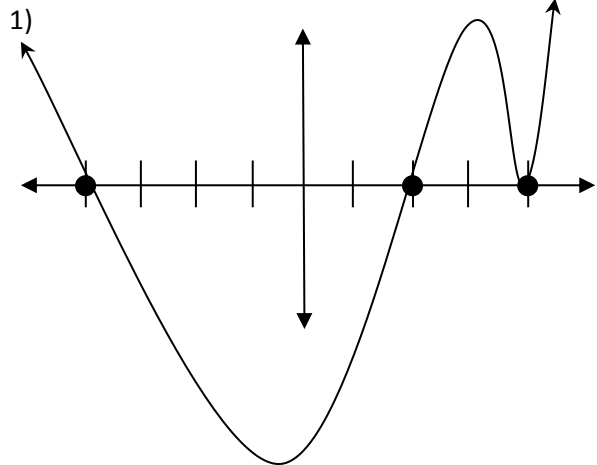


Sign analysis for  $(-7)$  reveals a positive answer, so the graph must start above the x-axis.



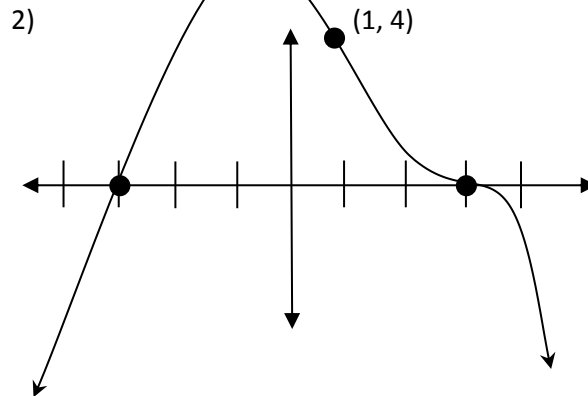
Sign analysis for  $(-4)$  reveals a negative answer, so the graph must start below the x-axis.

Given the graph, write one possible equation for the polynomial function.



$$y = \underline{(x + 4)(x - 2)(x - 4)^2}$$

Use sign analysis, plug in (-5) to see if the graph is correct.



$$y = \underline{-(x + 3)(x - 3)^3}$$

Using sign analysis at (-4) shows the graph to be correct only if the negative sign is added to the equation.

Now, for the graph from #2, using the given coordinates (1, 4), come up with the exact equation.

$$\begin{aligned}
 y &= a(x + 3)(x - 3)^3 \\
 4 &= a(1 + 3)(1 - 3)^3 \quad \text{plug in (1, 4)} \\
 4 &= a(4)(-2)^3 \\
 4 &= a(4)(-8) \\
 4 &= -32a \\
 a &= -1/8 \\
 y &= -1/8(x + 3)(x - 3)^3
 \end{aligned}$$

Homework: pg66-68 21-24, 27-32