COLLEGE REVIEW MATH — "MAXIMUM" WORD PROBLEMS

SECTION 2D

Solve the following problems by:

(1) using a chart if necessary

(2) writing a quadratic function to represent the given information

(3) using your graphing calculator or formula to find the "maximum" answer — formula: $x = \frac{-b}{2a}$

EXAMPLES:

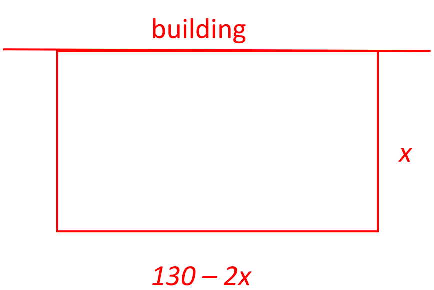
1) Find two numbers whose sum is 26 that has the maximum product.

First, choose a variable to represent one of the numbers, then determine how to use the "sum is 26" to represent the other. I like using a T-chart.

1^{st} #	2^{nd} #
x	26 – x

Product means to multiply, so…
f(x) = x(26 – x)
= 26x – x^2 or $-x^2 + 26x$
Using the formula from above…
x = -26/2(-1) or x = 13
Since the sum of the numbers is 26, one is 13, and the other is 13.

2) The owner of a business wishes to make a rectangular enclosure using the back of his building which measures 80 meters, along with the 130 meters of fence he has purchased. What dimensions should he use to get the maximum area?



width	length
x	*130-2x*

Since area = width X length…
f(x) = x(130 – 2x)
= 103x – $2x^2$ or $-2x^2 + 130x$
Using the formula…
x = -130/2(-2) or x = 32.5
The width = 32.5 meters
Length = 130 – 2(32.5) = 65 meters

3) A hot dog vendor sells an average of 160 hot dogs per day for $1.00 each. He wants to raise the price, but the other vendors warn him that for every nickel increase in price, he'll lose 5 sales. What should he set his new price at to maximize profits?

	#sales	price	profit
originally:	160	$1	$160
after price increase:	160 – 5n	$(1 + .05n)	(160 – 5n)(1 + .05n)

f(x) = (160 – 5n)(1 + .05n)
= 160 + 8n – 5n – $.25n^2$
= $-.25n^2 + 3n + 160$
Using the formula…
x = -3/2(-.25) or x = 6 (nickels)
The new price to maximize profit is: $1.30

NON-WORD PROBLEMS

FOR QUADRATIC FUNCTIONS:

To determine whether the equation has a maximum or minimum, find the sign for the quadratic term. Positive = minimum, Negative = maximum.

To find the maximum (or minimum) for "x", apply the formula $x = \frac{-b}{2a}$

To find the "y", plug in the x-value and compute.

EXAMPLES:

1) $f(x) = 4x^2 - 16x + 9$

max or min? min

max or min value of x? x = 2

y-value? y = -7

$x = -(-16)/2(4)$
$= 16/8$
$= 2$; plug in for y
$y = 4(2)^2 - 16(2) + 9$
$= 4(4) - 32 + 9$
$= -7$

2) $f(x) = 11 - x^2 + 8x$

max or min? max

max or min value of x? x = 4

y-value? y = 27

$x = -8/2(-1)$
$= -8/-2$
$= 4$
$y = 11 - 4^2 + 8(4)$
$= 11 - 16 + 32$
$= 27$

3) $f(x) = 2(x - 5)(x + 10)$

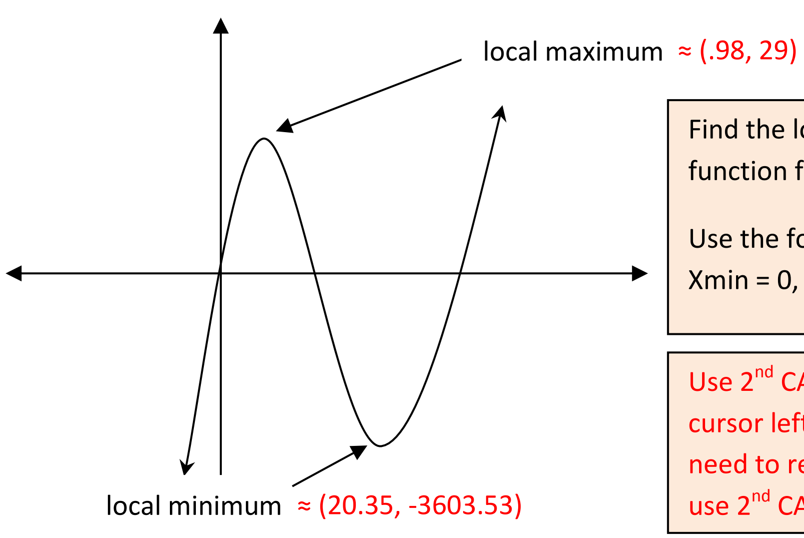
max or min? min

max or min value of x? x = -2.5

y-value? y = -112.5

$f(x) = 2(x^2 + 10x - 5x - 50)$
$= 2x^2 + 10x - 100$
$x = -10/2(2)$
$= -2.5$
$y = 2(-2.5 - 5)(-2.5 + 10)$
$= 2(-7.5)(7.5)$
$= -112.5$

LOCAL MAXIMUMS AND MINIMUMS



Find the local maximum and minimum for the function $f(x) = x^3 - 32x^2 + 60x$

Use the following WINDOW settings:
Xmin = 0, Xmax = 10, Ymin = -20, Ymax = 40

Use 2nd CALC, 4:maximum, positioning the cursor left, then right of the max point. You will need to reset the WINDOW to see the bottom, use 2nd CALC, 3:minimum to find the min.