College Review Math
Section 2F

SOLVING POLYNOMIAL EQUATIONS BY FACTORING

When in doubt...

1) $2x^5 - 22x^3 = 0$

$2x^3(x^2 - 11) = 0$
$x = 0, \quad x^2 - 11 = 0$ ← or run program... $x = \pm 3.4$
$x^2 = 11$
$x = \pm\sqrt{11}$

2) $3x^3 + 5x^2 - 12x - 20 = 0$

$3x^3 - 12x + 5x^2 - 20 = 0$
$3x(x^2 - 4) + 5(x^2 - 4) = 0$ ← factor by grouping
$(x^2 - 4)(3x + 5) = 0$
$(x - 2)(x + 2)(3x + 5) = 0$
$x = 2, x = -2, x = -5/3$

3) $3y^5 + 9y^3 + 24y^2 = -72$

$3y^5 + 9y^3 + 24y^2 + 72 = 0$
$y^5 + 3y^3 + 8y^2 + 24 = 0$
$y^3(y^2 + 3) + 8(y^2 + 3) = 0$
$(y^3 + 8)(y^2 + 3) = 0$
$(y + 2)(y^2 - 2y + 4)(y^2 + 3) = 0$
$y = -2$, run program twice → $y = 1 \pm 1.7i$, $y = \pm 1.7i$ or use quadratic formula by hand

4) $x^4 + 4x^2 - 3x^3 - 12x + 2x^2 + 8 = 0$

$x^2(x^2 + 4) - 3x(x^2 + 4) + 2(x^2 + 4) = 0$
$(x^2 - 3x + 2)(x^2 + 4) = 0$
$(x - 2)(x - 1)(x^2 + 4) = 0$
$x = 2, x = 1, \quad x^2 + 4 = 0$ ← or run program
$x^2 = -4$
$x = \pm 2i$

Quadratic Substitution (Quadratic Form)

Question: Why is the problem $4x^2 - 10x - 15 = 0$ easy to do?

It's a quadratic equation which can be factored or solved using the quadratic formula.

With that in mind...

5) $n^4 - 16n^2 + 63 = 0$

$x^2 - 16x + 63 = 0$
$(x - 9)(x - 7) = 0$
$x = 9, \quad x = 7$
$n^2 = 9$ and $n^2 = 7$
$n = \pm 3, n = \pm\sqrt{7}$

6) $6y^4 + 17y^2 = 14$ **with approximations

$6x^2 + 17x - 14 = 0$
$(3x - 2)(2x + 7) = 0$
$x = 2/3, \quad x = -7/2$
$y^2 = 2/3, \quad y^2 = -7/2$
$y = \pm .8, y = \pm 1.9i$

The final answers have been rounded. Should you choose to use the quad-program at any point, the solutions will not be 100% accurate.

7) $x^6 + 7x^3 - 8 = 0$

$z^2 + 7z - 8 = 0$
$(z + 8)(z - 1) = 0$
$z = -8, \quad z = 1$
$x^3 = -8$ and $x^3 = 1$
$x^3 + 8 = 0, x^3 - 1 = 0$
$(x+2)(x^2-2x+4) = 0, \ (x-1)(x^2+x+1) = 0$
$x = -2, x = 1 \pm 1.7i, \quad x = 1, x = -.5 \pm .9i$

8) $z^4 + 3z^2 - 54 = 0$ **likely to see on a quiz

$x^2 + 3x - 54 = 0$
$(x + 9)(x - 6) = 0$
$x = -9, \quad x = 6$
$z^2 = -9$ and $z^2 = 6$
$z = \pm 3i, z = \pm\sqrt{6}$

MIND YOUR P'S & Q'S

Find all the zeros (roots, solutions, answers) to the following polynomial equations, but first answer the questions.

1) $3x^3 + 13x^2 - 32x - 12 = 0$

Can you group it? Does have 4-parts, but will not group.

Can you use quadratic substitution? No, too many parts.

Then what do you do?

Construct a p & q-factor list, and choose a "potential" root from the list. Using trial and error and synthetic division, find a number that gives you a remainder of zero, then…

Listing the "potential roots" using the **RATIONAL ROOT THEOREM**

Examples (list the p, q, and p/q's only – you do not need to solve the equation)

2) $1x^7 - 3x^4 + 9x^3 + x + 32 = 0$

q-factor

p-factor

p: ±1,2,4,8,16,32

q: ±1

p/q: ±1,2,4,8,16,32

3) $9x^3 + x^2 + 8x - 4 = 0$

p: ±1,2,4

q: ±1,3,9

p/q: ±1,2,4,1/3,2/3,4/3,1/9,2/9,4/9

Examples: Use the rational root theorem to
find all zeros (real and imaginary) for each polynomial equation

4) $5x^3 - 3x^2 - 20x + 12 = 0$

5) $x^4 - 12x^3 + 37x^2 - 12x + 36 = 0$

Work for problems 4-6 are on the next page.

6) $3x^4 + 15x^3 + 10x^2 - 10x - 8 = 0$

4) $5x^3 - 3x^2 - 20x + 12 = 0$

p-factors: ±1,2,3,4,6,12

q-factors: ±1,5

p/q: ±1,2,3,4,6,12,1/5,2/5,3/5,4/5,6/5,12/5

2	5	-3	-20	12
		10	14	-12
	5	7	-6	0

$5x^2 + 7x - 6 = 0$

$(5x - 3)(x + 2) = 0$

$x = 3/5$, $x = -2$ and $x = 2$

5) $x^4 - 12x^3 + 37x^2 - 12x + 36 = 0$

p-factors: ±1,2,3,4,6,9,12,18,36

q-factors: ±1

p/q: will be exactly the same as the p-factors

6	1	-12	37	-12	36
		6	-36	6	-36
	1	-6	1	-6	0

The problem could also be finished using factoring by grouping.

6	1	-6	1	-6
		6	0	6
	1	0	1	0

$x^2 + 1 = 0$

$x = -1$

$x = \pm i$, $x = 6$

Since x = 6 is actually an answer twice (double root), it would be unnecessary to write it again.

6) $3x^4 + 15x^3 + 10x^2 - 10x - 8 = 0$

p-factors: ±1,2,4,8

q-factors: ±1,3

p/q: ±1,2,4,8,1/3,2/3,4/3,8/3

-4	3	15	10	-10	-8
		-12	-12	8	8
	3	3	-2	-2	0

$3x^3 + 3x^2 - 2x - 2 = 0$

$3x(x^2 + 1) - 2(x^2 + 1) = 0$

$(3x - 2)(x^2 + 1) = 0$

$X = 2/3$, $x = \pm i$