COLLEGE REVIEW MATH
SECTION 2G

GENERAL RESULTS FOR POLYNOMIAL EQUATIONS

(A) FIND THE COINCIDENCE

Solve the equation $x^2 + 5x - 24 = 0$ by factoring

$(x + 8)(x - 3) = 0$
$x = -8$ and $x = 3$

What is the sum of the roots? $-8 + 3 = -5$
What is the product of the roots? $(-8)(3) = -24$

Coincidence? The sum of the roots gives you the middle term (opposite sign), the product of the roots gives you the last term.

Solve the equation $6x^2 + 11x + 3 = 0$ by factoring

$(3x + 1)(2x + 3) = 0$
$x = -1/3$ and $x = -3/2$

What is the sum of the roots? $-1/3 + (-3/2) = -11/6$
What is the product of the roots? $(-1/3)(-3/2) = 3/6$

Coincidence? The same as the previous problem, but in addition, the denominator for both is the first term.

(B) APPLY THE COINCIDENCE

Find the sum & product of the roots for $5x^2 - 7x + 10 = 0$ without solving it!

Sum: $-(-7)/5 = 7/5$
Prod: $10/5 = 2$

Sum of the roots = $\frac{-b}{a}$

Product of the roots = $\frac{c}{a}$

(C) WRITE THE EQUATION GIVEN THE ROOTS BY USING THE SUM & PRODUCT

1) Roots: -3 and $\frac{2}{3}$

Sum: $-3 + 2/3 = \underline{\mathbf{-7/3}}$
Prod: $-3(2/3) = -2$
In order to find "a", the denominators must match so… Prod: $-2 = \underline{\mathbf{-6/3}}$
$3x^2 + 7x - 6 = 0$

2) Roots: $2 \pm \sqrt{5}$

Sum: $2 + \sqrt{5} + 2 - \sqrt{5}$
$= 4$ or $\underline{\mathbf{4/1}}$
Prod: $(2 + \sqrt{5})(2 - \sqrt{5})$
$= 4 - \sqrt{25}$
$= 4 - 5 = -1$ or $\underline{\mathbf{-1/1}}$
$x^2 - 4x - 1 = 0$

3) Roots: $1 \pm 3i$

Sum: $1 + 3i + 1 - 3i$
$= 2$ or $\underline{\mathbf{2/1}}$
Prod: $(1 + 3i)(1 - 3i)$
$= 1 - 9i^2$
$= 1 + 9 = 10$ or $\underline{\mathbf{10/1}}$
$x^2 - 2x + 10 = 0$

(D) WRITE THE CUBIC EQUATION GIVEN THREE ROOTS.

Roots: -2 and $3 \pm 2\sqrt{5}$

Sum: $3 + 2\sqrt{5} + 3 - 2\sqrt{5} = 6$ or $\underline{\mathbf{6/1}}$
Prod: $(3 + 2\sqrt{5})(3 - 2\sqrt{5})$
$= 9 - 4\sqrt{25} = 9 - 4(5)$
$= -11$ or $\underline{\mathbf{-11/1}}$
$x^2 - 6x + 11 = 0$

If (-2) is also a root, then the binomial: $(x + 2)$ represents it.

So, the cubic equation =
$(x + 2)(x^2 - 6x + 11)$
$= x^3 - 6x^2 + 11x + 2x^2 - 12x + 22$
$x^3 - 4x^2 + 23x + 22 = 0$

COMPLEX CONJUGATES THEROEM

Example: I'll give you two of the roots for the equation: $x^3 - 4x^2 + 30x - 52 = 0$

Roots: 2, 1 + 5i

Can you guess the third root? 1 – 5i

(A) USE THE COMPLEX CONJUGATES THEOREM TO NAME ANOTHER ROOT.

1) -6 – 3i Another root: -6 + 3i

2) $8+2\sqrt{7}$ Another root: $8 - 2\sqrt{7}$

(B) REVIEW (WITH A TWIST). USING THE SUM & PRODUCT OF ROOTS…

3) Find the quadratic equation with the root: $\frac{3-i\sqrt{5}}{4}$ → So, $\frac{3+i\sqrt{5}}{4}$ is the other root.

Sum: $\frac{3-i\sqrt{5}}{4} + \frac{3+i\sqrt{5}}{4} = 6/4$ Prod: $\frac{3-i\sqrt{5}}{4} \bullet \frac{3+i\sqrt{5}}{4} = \frac{9-i^2\sqrt{25}}{16} = \frac{9+5}{16} = 14/16$

The two fractions must be changed so that the denominators match. So…

Sum: **12/8** Prod: **7/8** (notice 12/8 reduces, but 7/8 does not, so this is the simplest possible solution)

Quadratic equation: $8x^2 - 12x + 7 = 0$

4) Find the quartic equation with roots: 3 + 2i & 4 + i

Sum: 3 + 2i + 3 – 2i = 6 or **6/1**

Prod: (3 + 2i)(3 – 2i) = $9 - 4i^2$ = 9 + 4 or **13/1**

Sum: 4 + i + 4 – i = 8 or **8/1**

Prod: (4 + i)(4 – i) = $16 - i^2$ = 16 + 1 or **17/1**

Now multiply the two equations together.

$(x^2 - 6x + 13)(x^2 - 8x + 17)$

$= x^4 - 8x^3 + 17x^2 - 6x^3 + 48x^2 - 102x + 13x^2$ - 104x + 221

Quartic equation: $x^4 - 14x^3 + 78x^2 - 206x + 221 = 0$

(C) REVERSING THE PROCESS. GIVEN AN EQUATION (AND WITHOUT FINDING THE ROOTS), FIND THE SUM & PRODUCT OF THE ROOTS.

5) $3x^2 - 12x + 8 = 0$

Sum: -(-12)/3 = 12/3 = 4

Prod: 8/3

6) $-4x^2 + 2x + 40 = 0$

Sum: -2/-1 = 1/2

Prod: 40/-4 = -10

Sum of the roots = $\frac{-b}{a}$

Product of the roots = $\frac{c}{a}$

(D) REFINING THE SUM & PRODUCT FORMULAS: GIVEN AN EQUATION WITH DEGREE HIGHER THAN 2, FIND THE SUM & PRODUCT OF THE ROOTS.

7) $x^3 - 4x^2 + 9x + 11 = 0$

Sum: -(-4)/1 = 4/1 = 4

Prod: -11/1 = -11

8) $9x^6 + 6x^5 + x^3 - 2x^2 + 1 = 0$

Sum: -6/9 = -2/3

Prod: 1/9

9) $5x^5 + 5x^3 - 3x^2 + x = 60$

$5x^5 + 0x^4 - 3x^2 + x - 60 = 0$

Sum: 0/5 = 0

Prod: 60/5 = 12

Sum of the roots = $\frac{-second\ term}{first\ term}$

Product of the roots =

even degree: $\frac{last\ term}{first\ term}$

odd degree: $\frac{-last\ term}{first\ term}$