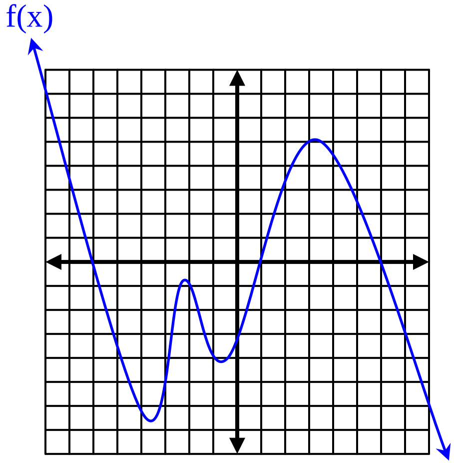
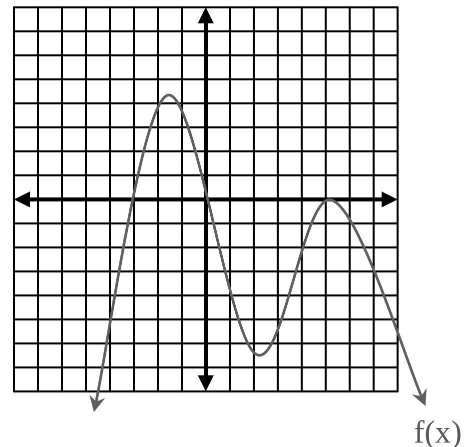
COLLEGE REVIEW MATH
SECTION 3B

POLYNOMIAL INEQUALITIES IN ONE VARIABLE

Use the graphs shown for each function to determine where $f(x) \geq 0$



Where is the graph above (>) or touching (=) the x-axis?



Solution set: $\{ x \leq -6 \text{ or } 1 \leq x \leq 6 \}$

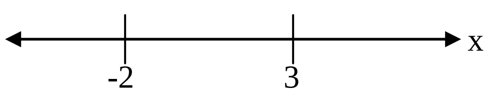
Solution set: $\{ -3 \leq x \leq 0 \text{ or } x = 5 \}$

SIGN ANALYSIS, AGAIN!

Solve each inequality using sign analysis. It wouldn't hurt to use the "x-axis" to help.

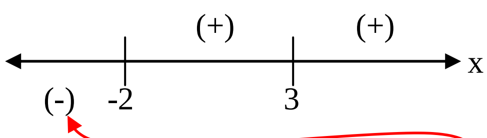
Example: $(x + 2)(x - 3)^2 < 0$

Roots = -2, 3



Check sign result for -3, 0, and 4

(-3): $(-3 + 2)(-3 - 3)^2$ or $(-)(+) = (-)$
(0): $(0 + 2)(0 - 3)^2$ or $(+)(+) = (+)$
(4): $(4 + 2)(4 - 3)^2$ or $(+)(+) = (+)$

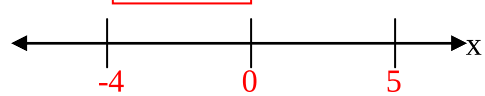


Solution set (where is it < 0)

$x < -2$

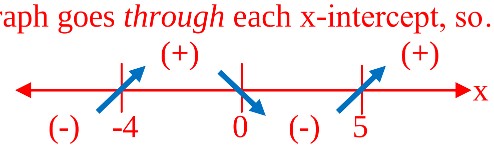
Your turn: $x(x - 5)(x + 4) \geq 0$

Roots = $\boxed{0, 5, -4}$



Check sign result for -5, then apply the "short cut" (shown below)
(-5): $-5(-5 - 5)(-5 + 4)$ or $(-)(-)(-) = (-)$

Short cut: since all the roots to the problem are single roots, we know that the graph goes *through* each x-intercept, so…



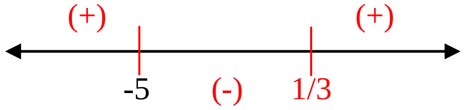
Solution set: $\{ -4 \leq x \leq 0 \text{ or } x \geq 5 \}$

Without the roots in front of your face, what can you do? Factor it out!

1) $3x^2 + 14x - 5 \le 0$

$(3x - 1)(x + 5) \le 0$
Roots: 1/3, -5

Sign analysis for (-6): (-)(-) = (+)

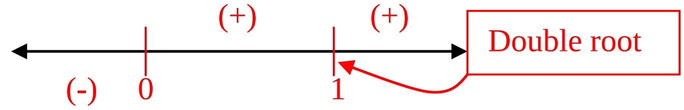


Solution set: $\{ -5 \le x \le 1/3 \}$

2) $x^3 - 2x^2 + x > 0$

$x(x^2 - 2x + 1) > 0$
$x(x - 1)(x - 1) > 0$
$x(x - 1)^2 > 0$
Roots: 0, 1

Sign analysis for (-1): (-)(+) = (-)

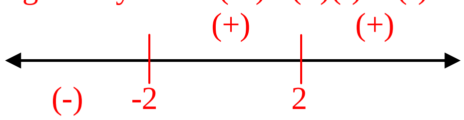


Solution set: $\{ x > 0,\ x \ne 1 \}$

3) $x^3 - 2x^2 - 4x + 8 < 0$

$x^2(x - 2) - 4(x - 2) < 0$
$(x - 2)(x^2 - 4) < 0$
$(x - 2)(x - 2)(x + 2) < 0$
$(x - 2)^2(x + 2) < 0$
Roots: 2, -2

Sign analysis for (-3): (+)(-) = (-)



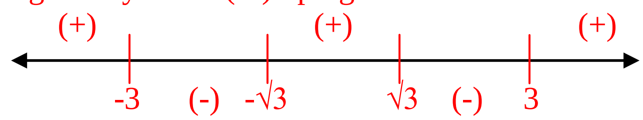
Solution set: $\{ x < -2 \}$

4) $x^4 - 12x^2 \ge -27$

$x^4 - 12x^2 + 27 \ge 0$
quad. substitution
$x^2 = 9$ or $x^2 = 3$
$x = \pm 3$ or $x = \pm\sqrt{3}$

$y^2 - 12y + 27$
$(y - 9)(y - 3)$
$y = 9 \quad y = 3$

Sign analysis for (-4): plug in here = +91



Solution set: $\{ x \le -3 \text{ or } -\sqrt{3} \le x \le \sqrt{3} \text{ or } x \ge 3 \}$

5) $4x^3 + 11x^2 + 5x - 2 > 0$

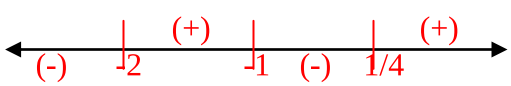
p's & q's to get this one!
p-factors: ±1, 2
q-factors: ±1, 2, 4
p/q: ±1, 2, 1/2, 1/4

synthetic division:

$\boxed{-1}$	4	11	5	-2
		-4	-7	2
	4	7	-2	0

$4x^2 + 7x - 2 = 0$
$(4x - 1)(x + 2) = 0$
$x = 1/4$, -2 and -1

Sign analysis for (-3): plug into original problem = -26



Solution set: $\{ -2 < x < -1 \text{ or } x > 1/4 \}$

RATIONAL INEQUALITIES

<u>Restrictions</u> (or excluded values) are numbers that cannot be substituted into an equation or inequality.

Name the restrictions for the following inequalities. Remember, when in doubt…

1) $\dfrac{3x+8}{(x-4)(x+9)} \le 0$

x ≠ 4, -9

2) $\dfrac{x-7}{5x^2-25x} > 0$

$\boxed{5x(x-5)}$

x ≠ 0, 5

3) $\dfrac{11}{y^2-15y+54} \ge 0$

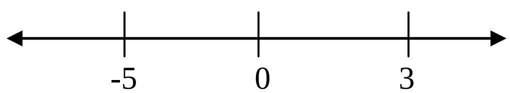
$\boxed{(y-6)(y-9)}$

y ≠ 6, 9

Solving rational inequalities

Use the roots (from the numerator) and restrictions (from the denominator) to use sign analysis.

Example: $\dfrac{(x-3)^2}{x(x+5)} \le 0$



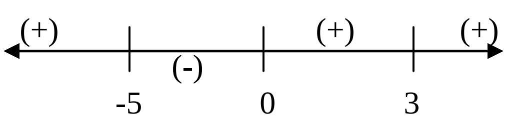
sign analysis

-6: (+)/(-)(-) = (+)

-1: (+)/(-)(+) = (-)

1: (+)/(+)(+) = (+)

4: (+)/(+)(+) = (+)



Solution set? ____________________

1) $\dfrac{x}{(x-6)(x+1)} > 0$

2) $\dfrac{(2x-3)(x-7)}{x^2} \ge 0$

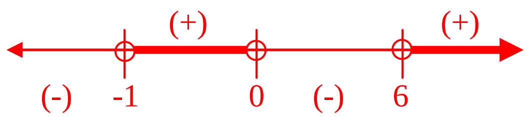
3) $\dfrac{x^2-16}{3x-24} \le 0$

Solutions to #1-3 are on the next page.

1) $\dfrac{x}{(x-6)(x+1)} > 0$

Roots: x = 0
Restrictions (undefined values): x = 6, -1

For your number line graph, it may pay to do this: restrictions are always values that *cannot* be used even if the problem is $\geq$ or $\leq$, so placing open circles on the graph for undefined values, and closed circles for roots that are $\geq$ or $\leq$ might help writing the solution set. Since ***this*** problem is > and not $\geq$, all the circles will be open.



You may want to shade the areas that are applicable as well. Since the original problems says "greater than", we are looking for values that are (+).

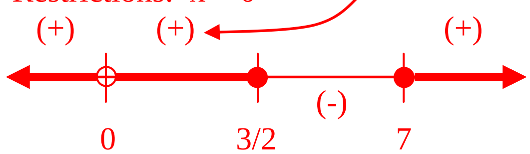
Sign analysis for (-2): (-) / (-)(-) = (-)

<u>Solution set: { -1 < x < 0 or x > 6 }</u>

2) $\dfrac{(2x-3)(x-7)}{x^2} \geq 0$

Roots: x = 3/2, 7
Restrictions: x = 0

Since 0 is a "double" restriction, the graph will not go through the x-intercept, but stay above the x-axis instead.

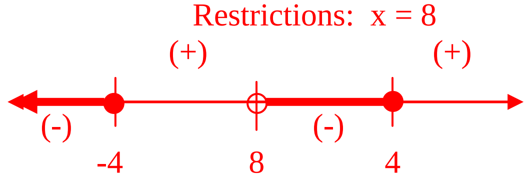


Sign analysis for (-1): (-)(-) / (+) = (+)

<u>Solution set: { x < 3/2 or x > 7, x ≠ 0 }</u>

3) $\dfrac{x^2-16}{3x-24} \leq 0$ $\boxed{\dfrac{(x-4)(x+4)}{3(x-8)} \leq 0}$

Roots: x = 4, -4
Restrictions: x = 8



Sign analysis for (-5) using factored version of problem: (-)(-) / (-) = (-)

<u>Solution set: { x < -4 or 8 < x < 4 }</u>