

Note: split problems up and use multiple graphs of this same series of functions

Find: 1) $g(2) - h(2) = \underline{1 - (-3) = 4}$

2) $f(4) + g(4) = \underline{-1 + 1 = 0}$

Where is the graph of $g(x)$ above $f(x)$?

3) max value of $f(x) - h(x)$ 5

4) values of x where $g(x) - f(x) > 0$ $-3 < x < -2$ or $2 < x < 6$

5) x-values where...

6) min value of $h(x) - g(x)$ -4

where $f(x) = g(x)$ -2, 2, 6

7) **Draw** $f(x) - 2$ See above (in red)

where $f(x) = h(x)$ 4

Where is the graph of $f(x)$ the furthest above $h(x)$? Then subtract the y-values.

Where is the graph of $h(x)$ the furthest below $g(x)$? Then subtract the y-values.

where $g(x) = h(x)$ 5

Find their intersection points.

Use the following functions for the next section.

$$f(x) = x^2 - 16 \quad g(x) = 2x - 8 \quad h(x) = x^3 + 1$$

1) $(f + g)(x)$

$$\begin{aligned} (x^2 - 16) + (2x - 8) \\ = x^2 + 2x - 16 - 8 \\ = x^2 + 2x - 24 \\ (f + g)(x) = x^2 + 2x - 24 \end{aligned}$$

2) $(h - g)(x)$

$$\begin{aligned} (x^3 + 1) - (2x - 8) \\ = x^3 + 1 - 2x + 8 \\ = x^3 - 2x + 9 \\ (h - g)(x) = x^3 - 2x + 9 \end{aligned}$$

3) $(f \cdot h)(x)$

$$\begin{aligned} (x^2 - 16)(x^3 + 1) \\ = x^5 + x^2 - 16x^3 - 16 \\ = x^5 - 16x^3 + x^2 - 16 \\ (f \cdot h)(x) = x^5 - 16x^3 + x^2 - 16 \end{aligned}$$

4) $\left(\frac{f}{g}\right)(x)$

$$\begin{aligned} \frac{x^2 - 16}{2x - 8} \\ = \frac{(x - 4)(x + 4)}{2(x - 4)} \\ = \frac{x + 4}{2} \\ \left(\frac{f}{g}\right)(x) = \frac{x + 4}{2} \end{aligned}$$

COMPOSITION OF FUNCTIONS

Use the following functions for the next section.

$$f(x) = 2x - 5 \quad g(x) = \sqrt{x - 3} \quad h(x) = 4x^2 + 3$$

1) $f(g(h(-3)))$

$$\begin{aligned} h(-3) &= 4(-3)^2 + 3 \\ &= 39 \\ g(39) &= \sqrt{39 - 3} \\ &= 6 \\ f(6) &= 2(6) - 5 = 7 \\ f(g(h(-3))) &= 7 \end{aligned}$$

2) $h(f(g(19)))$

$$\begin{aligned} g(19) &= \sqrt{19 - 3} \\ &= 4 \\ f(4) &= 2(4) - 5 \\ &= 3 \\ h(3) &= 4(3)^2 + 3 = 39 \\ h(f(g(19))) &= 39 \end{aligned}$$

3) $h(g(f(2)))$

$$\begin{aligned} f(2) &= 2(2) - 5 \\ &= -1 \\ f(-1) &= \sqrt{-1 - 3} \\ &= 2i \\ h(2i) &= 4(2i)^2 + 3 = -16 + 3 = -13 \\ h(g(f(2))) &= -13 \end{aligned}$$

4) $f(g(h(x)))$

$$\begin{aligned} g(h(x)) &= \sqrt{4x^2 + 3 - 3} \\ &= \sqrt{4x^2} \\ &= 2x \\ f(g(h(x))) &= 2(2x) - 5 \\ &= 4x - 5 \end{aligned}$$

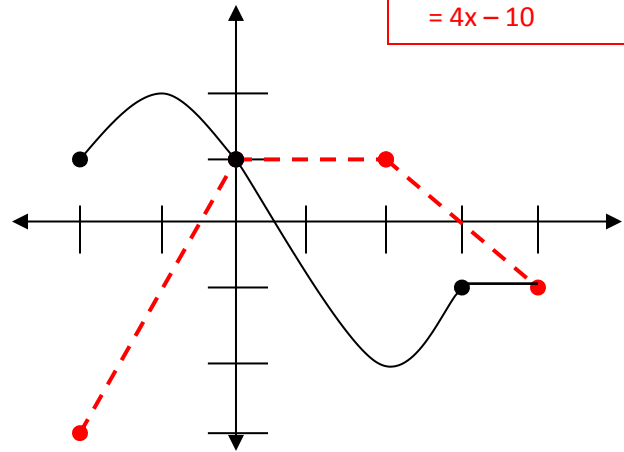
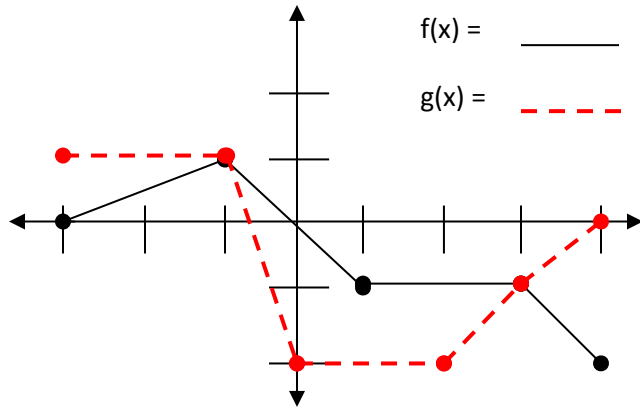
5) $f(h(g(x)))$

$$\begin{aligned} h(g(x)) &= 4(\sqrt{x - 3})^2 + 3 \\ &= 4(x - 3) + 3 \\ &= 4x - 12 + 3 \\ &= 4x - 9 \\ f(h(g(x))) &= 2(4x - 9) - 5 \\ &= 8x - 18 - 5 \\ &= 8x - 23 \end{aligned}$$

6) $g(h(f(x)))$

$$\begin{aligned} h(f(x)) &= 4(2x - 5)^2 + 3 \\ &= 4(2x - 5)(2x - 5) + 3 \\ &= 4(4x^2 - 20x + 25) + 3 \\ &= 16x^2 - 80x + 100 + 3 \\ &= 16x^2 - 80x + 103 \\ g(h(f(x))) &= \sqrt{(16x^2 - 80x + 103) - 3} \\ &\text{continued on next page} \end{aligned}$$

More practice.



$$\begin{aligned}
 6) &= \sqrt{16x^2 - 80x + 100} \\
 &= \sqrt{(4x - 10)(4x - 10)} \\
 &= 4x - 10
 \end{aligned}$$

- 1) $f(1) + g(2) = \underline{-1 + (-2) = -3}$
 - 2) $g(-3) - f(4) = \underline{1 - (-2) = 3}$
- Where is $f(x) - g(x)$...
- 3) zero? @x = -1, 3
 - 4) positive? -1 < x < 3
 - 5) negative? -3 < x < -1 or 3 < x < 4

- 1) $f(4) - g(4) = \underline{-1 - (-1) = 0}$
 - 2) $g(2) + f(3) = \underline{1 + (-1) = 0}$
- 3) The minimum value of $f(x) - g(x)$?
 $-2 - 1 = -3$
 occurs @x = 2
- 4) The maximum value of $f(x) - g(x)$?
 $1 - (-3) = 4$
 occurs @x = -2
- 5) Values of x where $f(x) = g(x)$ @x = 0, 4