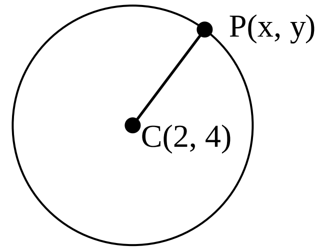
SECTION 6A

EQUATIONS OF CIRCLES

Given the radius and center of a circle as in the one shown to the right, we can find its equation using the distance formula



Assume PC = 5

$\sqrt{(x-2)^2+(y-4)^2} = 5$

by squaring both sides

$(x - 2)^2 + (y - 4)^2 = 25$

or the general form; $\mathbf{(x - h)^2 + (y - k)^2 = r^2}$ where the center = (h,k) and the radius = r

Examples:

Determine the center and radius of each circle whose equation is given.

1) $x^2 + y^2 = 81$

$(x + 0)^2 + (y + 0)^2 = 9^2$

C = (0, 0); r = 9

2) $(x + 5)^2 + y^2 = 4$

$(x + 5)^2 + (y + 0)^2 = 2^2$

C = (-5, 0); r = 2

3) $x^2 + (y - 9)^2 = 45$

$(x + 0)^2 + (y - 9) = (\sqrt{45})^2$

C = (0, 0); $r = \sqrt{45}$

$= 3\sqrt{5}$

Write an equation for the circle described.

1) The center is (7, -6) and radius = 12

$(x - 7)^2 + (y + 6)^2 = 12^2$
$(x - 7)^2 + (y + 6)^2 = 144$

2) C(-1, 0), $r = \sqrt{11}$

$(x + 1)^2 + (y + 0)^2 = (\sqrt{11})^2$
$(x + 1)^2 + y^2 = 11$

3) The center is (-3, -2) and the circle passes through the point (1, 3)

$(x + 3)^2 + (y + 2)^2 = r^2$
Sub in (1, 3) for x & y.

$(1 + 3)^2 + (3 + 2)^2 = r^2$
16 + 25 = r^2

$r^2 = 41$
Answer: $(x + 3)^2 + (y + 2)^2 = 41$

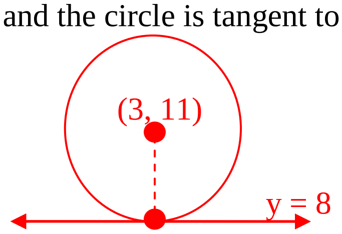
4) The endpoints of the diameter are (-5, 0) and (2, 5)

Use the midpt. formula to find the "center" $\left(\frac{-5+2}{2}, \frac{0+5}{2}\right) = (-1.5, 2.5)$

$(x + 1.5)^2 + (y - 2.5)^2 = r^2$, sub in either point.
$(-5 + 1.5)^2 + (0 - 2.5)^2 = r^2$
$18.5 = r^2$ Answer: $(x + 1.5)^2 + (y - 2.5)^2 = 18.5$

When is it a good idea to draw the object? (ALMOST ALWAYS)

5) The center is (3, 11) and the circle is tangent to the line y = 8



A line tangent to a circle is always perpendicular to the center, so the radius is just the distance (vertically) from y = 8 to the point (3, 11) or 3.
Answer: $(x - 3)^2 + (y - 11)^2 = 9$

MANIPULATING EQUATIONS

Do you remember how to complete the square?

a) $x^2 + 6x + \underline{\ 9\ }$ b) $y^2 - 14y + \underline{\ 49\ }$

Take 1/2 of the middle term (6 or -14), then square it.

What would be significant about putting these problems in perfect square form?

When you factor them, they look like $(\quad)^2$ which appears in most circle equations.

Examples:
Write each equation in center-radius form. $(x - h)^2 + (y - k)^2 = r^2$

1) $x^2 + y^2 + 10y - 4x = 10$

$x^2 - 4x \underline{\quad} + y^2 + 10y \underline{\quad} = 10$
$x^2 - 4x \underline{+4} + y^2 + 10y \underline{+25} = 10$
$\underline{+4+25}$
$(x - 2)(x - 2) + (y + 5)(y + 5) = 39$
$(x - 2)^2 + (y + 5)^2 = 39$

2) $x^2 - 14x + y^2 = 1$

$x^2 - 14x \underline{\quad} + y^2 = 1 \underline{\quad}$
$x^2 - 14x \underline{+49} = 1 \underline{+49}$
$(x - 7)(x - 7) + y^2 = 50$
$(x - 7)^2 + y^2 = 50$

3) $x^2 + y^2 + 12x + 5y + 7 = 0$

$x^2 + 12x \underline{\quad} + y^2 + 5y \underline{\quad} = -7$
$x^2 + 12x \underline{+36} + y^2 + 5y \underline{+6.25} = -7$
$+42.25$
$(x + 6)^2 + (y + 2.5)^2 = 35.25$

4) $2x^2 + 2y^2 - 16x - 8y = 2$

Div. by (2): $x^2 + y^2 - 8x - 4y = 1$
$x^2 - 8x \underline{\quad} + y^2 - 4y \underline{\quad} = 1 \underline{\quad}$
$x^2 - 8x \underline{+16} + y^2 - 4y \underline{+4} = 1 + \underline{16 + 4}$
$(x - 4)^2 + (y - 2)^2 = 21$

5) $3y^2 + 3x^2 - 18y + 3x - 5 = 0$

Div. by (3): $y^2 + x^2 - 6y + x - 5/3 = 0$
$x^2 + x \underline{\quad} + y^2 - 6y \underline{\quad} = 5/3 \underline{\quad}$
$x^2 + x + \underline{1/4} + y^2 - 6y \underline{+9} = 5/3 \underline{+1/4+9}$
$(x + 1/2)^2 + (y - 3)^2 = 131/12$

SOLVING SYSTEMS OF EQUATIONS

Do you remember how to solve a system of equations?

Find where these two lines intersect: $x - y = 9$
$3x + 2y = 2$

Substitution method: solve for x (top equation): $x = y + 9$
Substitute $(y + 9)$ in for x (bottom equation): $3(y + 9) + 2y = 2$
Solve for y: $3y + 27 + 2y = 2$
$5y = -25$
$y = -5$
Sub $y = -5$ back into either original equation: $x - (-5) = 9$
$x = 4$
Answer: $(4, -5)$

Examples:
Find the points of intersection. If the graphs are tangent or do not intersect state so.

1) $y = 2x - 2$ and
$x^2 + y^2 = 25$

$x^2 + (2x - 2)^2 = 25$
$x^2 + (2x - 2)(2x - 2) = 25$
$x^2 + 4x^2 - 4x - 4x + 4 = 25$
$5x^2 - 8x - 21 = 0$
$\underline{x = 3,\ x = -1.4;}$ plug both in:
$y = 2(3) - 2$ $y = 2(-1.4) - 2$
$y = 4$ $y = -4.8$
Answers: $(3, 4)$ & $(-1.4, -4.8)$

2) $y + 3x = 10$ and
$x^2 + y^2 = 4$

$y = -3x + 10$

$x^2 + (-3x + 10)^2 = 4$
$x^2 + (-3x + 10)(-3x + 10) = 4$
$x^2 + 9x^2 - 30x - 30x + 100 = 4$
$10x^2 - 60x + 96 = 0$
$\underline{5x^2 - 30x + 48 = 0}$
The solutions are imaginary.
Therefore, the graphs of the line and circle *do not intersect*. Ø

3) $y = 2x$ and
$x^2 + y^2 + 2x - 6y + 5 = 0$

$x^2 + (2x)^2 + 2x - 6(2x) + 5 = 0$
$x^2 + 4x^2 + 2x - 12x + 5 = 0$
$5x^2 - 10x + 5 = 0$
$x^2 - 2x + 1 = 0$
$\underline{x = 1}$ (only); plug in:
$y = 2(1)$
$y = 2$
Answer: $(1, 2)$

Since there is just one answer, they are *tangent*.