SECTION 6B

ELLIPSES

Generic form for ellipses (not centered at the origin):

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad \text{or} \quad \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$

where a > b; Center = (h, k)

Examples: (find the center, "a", "b" and "c")

1) $\frac{(x+2)^2}{100}+\frac{(y-5)^2}{64}=1$

C = (-2, 5)

a = 10

b = 8

c = 6

$c^2 = a^2 - b^2$
$c^2 = 100 - 64$
$c^2 = 36$
$c = 6$

2) $\frac{x^2}{20}+\frac{(y+3)^2}{25}=1$

C = (0, -3)

a = 5

b = $2\sqrt{5}$

c = $\sqrt{5}$

$c^2 = a^2 - b^2$
$c^2 = 25 - 20$
$c^2 = 5$
$c = \sqrt{5}$

3) $9(x + 4)^2 + 2(y + 5)^2 = 36$

C = (-4, -5)

a = $3\sqrt{2}$

b = 2

c = 4

$$\frac{9(x+4)}{36}+\frac{2(y+5)}{36}=1$$
$$\frac{(x+4)}{4}+\frac{(y+5)}{18}=1$$

$c^2 = 18 - 4$
$c^2 = 16$
$c = 4$

4) $9x^2 - 18x + 16y^2 - 64y = 71$

Div. by 9: $x^2 - 2x$ ___ $+ 16/9y^2 - 64/9y = 71/9$ ___

$x^2 - 2x \underline{+1} + 16/9y^2 - 64/9y = 71/9 \underline{+1}$

$(x - 1)^2 + 16/9y^2 - 64/9y = 80/9$

Mult by 9/16: $\frac{9(x-1)^2}{16} + y^2 - 4y$ ___ $= 5$

$\frac{9(x-1)^2}{16} + y^2 - 4y \underline{+4} = 5 \underline{+4}$

$\frac{9(x-1)^2}{16} + (y - 2)^2 = 9$

Div. by 9: $\frac{(x-1)^2}{16} + \frac{(y-2)^2}{9} = 1$

SKETCHING THE ELLISPSE: Keep in mind the center is no longer located at the origin, therefore, the major and minor axes (distances a & b) must be counted from the center (left, right, up & down)

Examples: Sketch #1 & #2 from above. Find the coordinates for each vertex and focus.

#1) C(-2,5)

a=10

b=8

c=6

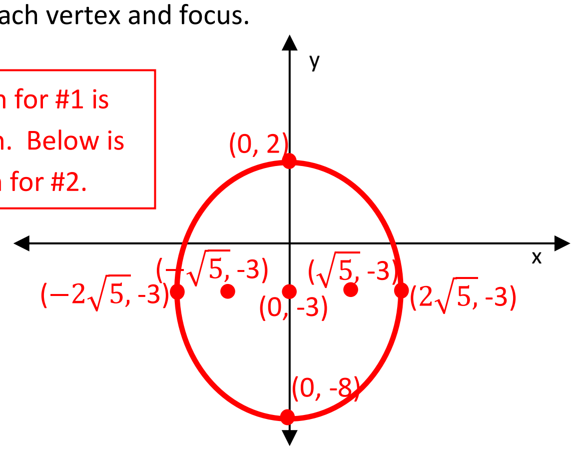
#2) C(0,-3)

a=5

b= $2\sqrt{5}$

c= $\sqrt{5}$

The sketch for #1 is not shown. Below is the sketch for #2.



Directions: Use the information given to find the equation for the ellipse described for each problem.

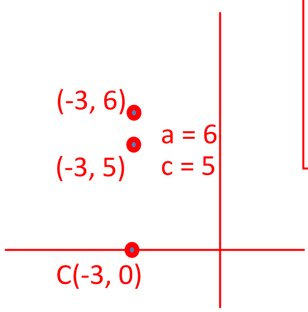
To write the equation for an ellipse (not centered at the origin), the KEY is to know three things; the CENTER, “a” and “b”.

Examples:

1) Center = (-3, 0)
Focus = (-3, 5)
Vertex = (-3, 6)

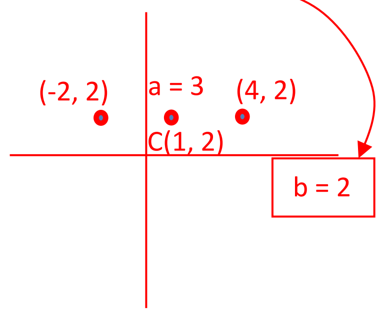
$c^2 = a^2 - b^2$
$5^2 = 6^2 - b^2$
$25 = 36 - b^2$
$b^2 = 11$

$$\frac{(x+3)^2}{11} + \frac{y^2}{36} = 1$$

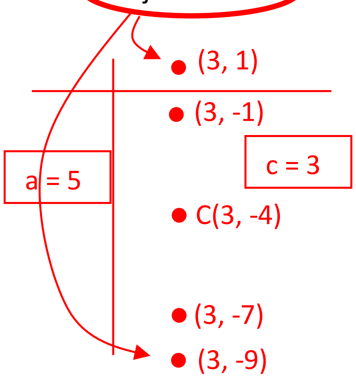


2) Vertices are: (4, 2) and (-2, 2)
Minor axis is 4 units long

$$\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$$



3) Foci are: (3, -1) and (3, -7)
Major axis is 10 units long



$c^2 = a^2 - b^2$
$3^2 = 5^2 - b^2$
$9 = 25 - b^2$
$b^2 = 16$

$$\frac{(x-3)^2}{16} + \frac{(y+4)^2}{25} = 1$$

4) Center = (-6, 3)
Vertex = (1, 3)
Tangent to the x-axis

$$\frac{(x+6)^2}{49} + \frac{(y-3)^2}{9} = 1$$