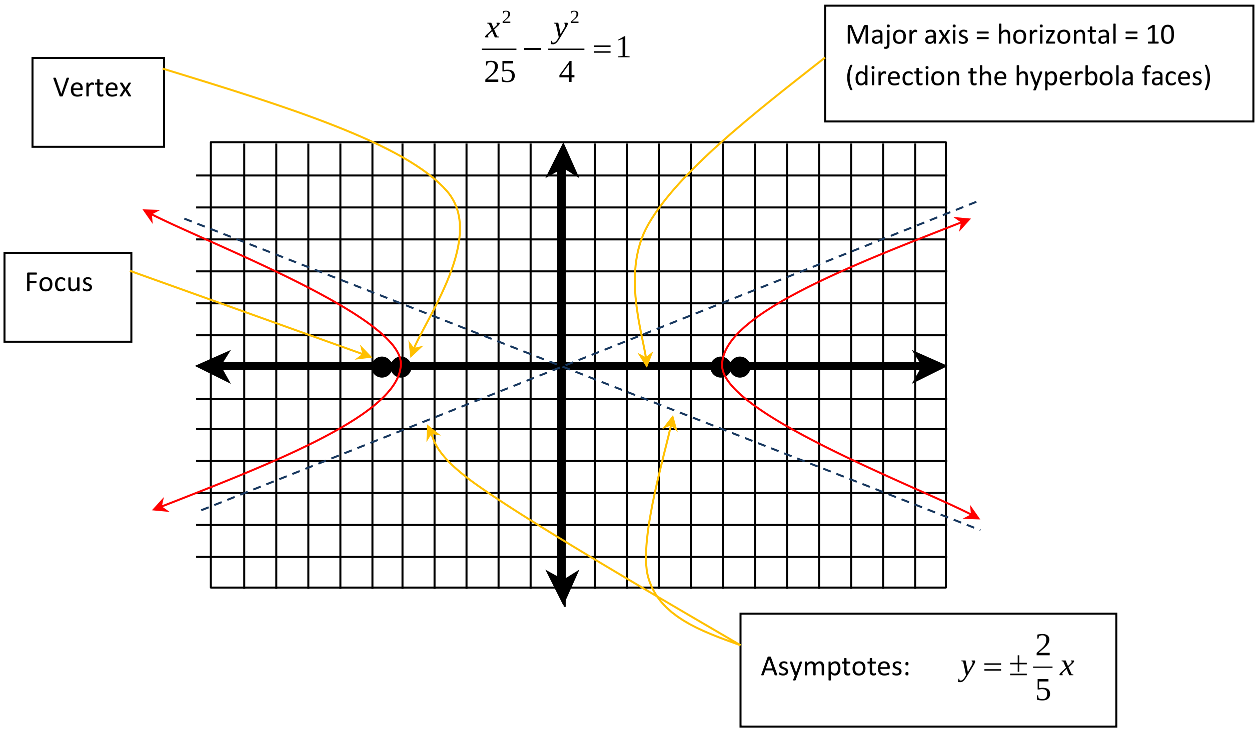
College Review Math

Section 6C

HYPERBOLAS

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$



Hyperbola generic form: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\qquad$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

a is not necessarily greater than ***b***

Asymptotes: $y = \pm\frac{b}{a}$ $\qquad$ $y = \pm\frac{a}{b}$

Foci: $c^2 = a^2 + b^2$

Div. by 100:

$\frac{25x^2}{100} - \frac{4y^2}{100} = 1$

$\frac{x^2}{4} - \frac{y^2}{25} = 1$

Examples:

For the following equations, find a, b, c and the equations for the asymptotes. Also determine if the major axis is horizontal or vertical.

1) $\frac{y^2}{36} - \frac{x^2}{49} = 1$

a = 6

b = 7

c = $\sqrt{85}$

$c^2 = a^2 + b^2$
$c^2 = 36 + 49$
$c^2 = 85$
$c = \sqrt{85}$

Asym: $y = \pm\frac{6}{7}x$

Major axis: Vert.

2) $\frac{x^2}{16} - \frac{y^2}{8} = 1$

a = 4

b = $2\sqrt{2}$

c = $2\sqrt{6}$

$c^2 = a^2 + b^2$
$c^2 = 16 + 8$
$c^2 = 24$
$c = 2\sqrt{6}$

Asym: $y = \pm\frac{\sqrt{2}}{2}x$

Major axis: Horiz.

3) $25x^2 - 4y^2 = 100$

a = 2

b = 5

c = $\sqrt{29}$

$c^2 = a^2 + b^2$
$c^2 = 4 + 25$
$c^2 = 29$
$c = \sqrt{29}$

Asym: $y = \pm\frac{5}{2}x$

Major axis: Horiz.

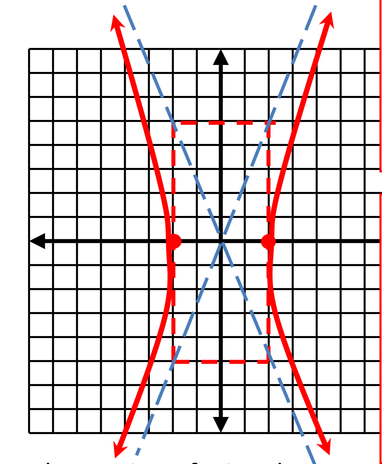
Sketching a hyperbola (using the box). Make a box with dimensions a X b.

Examples:

1) $\frac{x^2}{4} - \frac{y^2}{25} = 1$

a = 2
b = 5

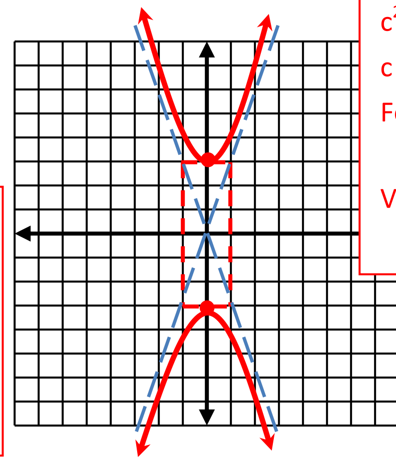
While the asymptotes (in blue) are part of the graph, the box (in red) technically is not.



$c^2 = a^2 + b^2$
$c^2 = 4 + 25$
$c^2 = 29$
$c = \sqrt{29}$
Foci: $(\sqrt{29}, 0)$ & $(-\sqrt{29}, 0)$
Vertices: (2, 0) & (-2, 0)

2) $\frac{y^2}{9} - x^2 = 1$

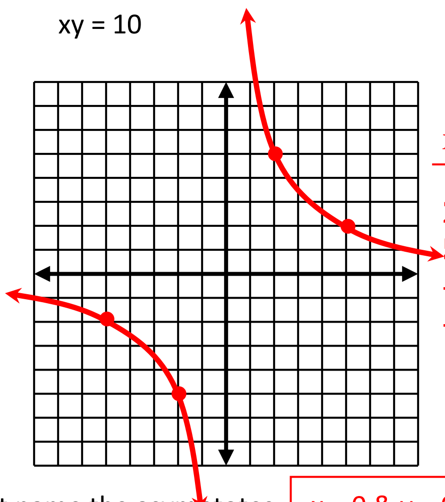
a = 3
b = 1



$c^2 = a^2 + b^2$
$c^2 = 9 + 1$
$c^2 = 10$
$c = \sqrt{10}$
Foci: $(0, \sqrt{10})$ & $(0, -\sqrt{10})$
Vertices: (0, 3) & (0, -3)

Name the vertices, foci and asymptotes for the previous sketches.

3) xy = 10

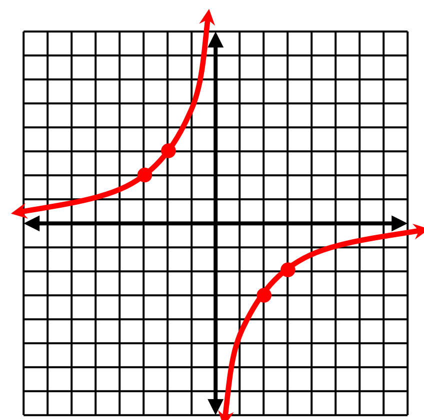


x	y
2	5
5	2
-2	-5
-5	-2

Just name the asymptotes.

x = 0 & y = 0 (the axes)

4) xy = -6



x	y
-2	3
-3	2
2	-3
3	-2

asymptotes: x = 0 & y = 0

Find the equation of the hyperbola with center at the origin that satisfies the given conditions.

Just need "a" and "b" and you've got it!

Examples:

1) Vertex: (0, -3) — a = 3

Focus: $(0, -3\sqrt{2})$ — $c = 3\sqrt{2}$

$c^2 = a^2 + b^2$
$(3\sqrt{2})^2 = 9 + b^2$
$18 = 9 + b^2$
$9 = b^2$

$$\frac{y^2}{9} - \frac{x^2}{9} = 1$$

2) Vertex: (8, 0) — a = 8

Asymptote: $y = \frac{5}{8}x$ — $= \frac{b}{a} = \frac{5}{8}$, so, b = 5

$$\frac{x^2}{64} - \frac{y^2}{25} = 1$$

3) Asymptote: $y = \frac{3}{2}x$ — $= \frac{a}{b}$, a=3, b=2

major axis: vertical

So, asymptotes are ±a/b

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

For the following equations, find the center, a, b, c and the slopes for the asymptotes. Also determine if the major axis is horizontal or vertical.

1) $\frac{(x+7)^2}{49} - \frac{(y+5)^2}{15} = 1$

Center = (-7, -5)

a = 7

b = $\sqrt{15}$

c = 8

$c^2 = a^2 + b^2$
$c^2 = 49 + 15$
$c^2 = 64$
c = 8

Asym. slopes: $\pm\frac{\sqrt{15}}{7}$

Major axis: Horiz.

2) $(y-2)^2 - 16x^2 = 16$

Div. by 16:
$\frac{(y-2)^2}{16} - x^2 = 1$

Center = (0, 2)

a = 4

b = 1

c = $\sqrt{17}$

$c^2 = a^2 + b^2$
$c^2 = 16 + 1$
$c^2 = 17$
c = $\sqrt{17}$

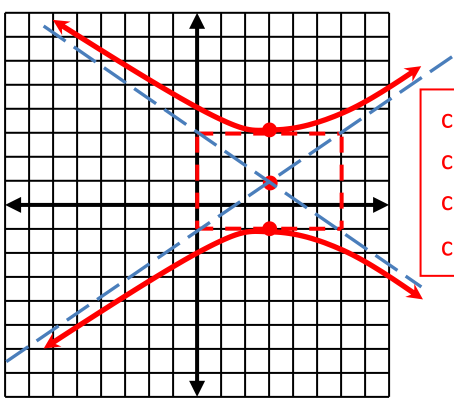
Asym. slopes: ±4

Major axis: Vert.

Sketching a hyperbola (using the box). Make a box with dimensions a X b. Name the vertices and foci for each.

Examples:

3) $\frac{(y-1)^2}{4} - \frac{(x-3)^2}{9} = 1$

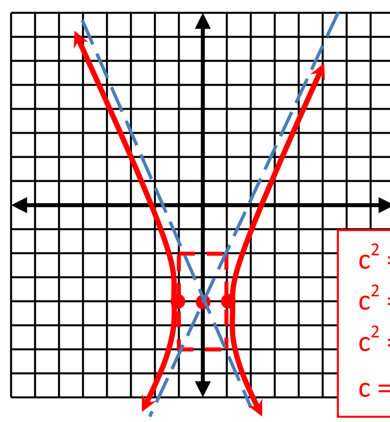


$c^2 = a^2 + b^2$
$c^2 = 4 + 9$
$c^2 = 13$
c = $\sqrt{13}$

Vertices: (3, 3) & (3,-1)

Foci: (3, 1+$\sqrt{13}$) & (3, 1− $\sqrt{13}$)

4) $x^2 - \frac{(y+4)^2}{4} = 1$



$c^2 = a^2 + b^2$
$c^2 = 1 + 4$
$c^2 = 5$
c = $\sqrt{5}$

Vertices: (-1, -4) & (1, -4)

Foci: (-$\sqrt{5}$, -4) & ($\sqrt{5}$, -4)

Find the equation for the hyperbola (not centered at the origin) described.

Sketches for #5-7 appear on the next page.

5) Center (-1, -5)
Vertex (-1, -8)

Focus (-1, -10)

$\frac{(y+5)^2}{9} - \frac{(x+1)^2}{16} = 1$

6) Vertices are
(0, 2) & (8, 2)

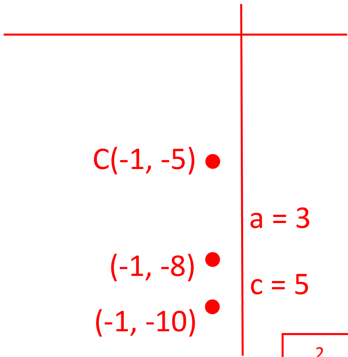
Asym. slopes = $\pm\frac{3}{2}$

$\frac{(x-4)^2}{16} - \frac{(y-2)^2}{36} = 1$

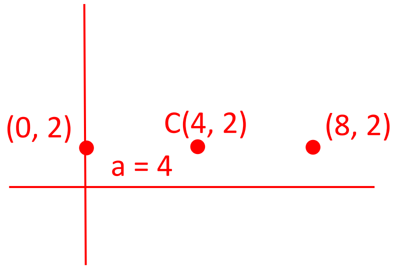
7) Foci are
(-3, 4) & (-3, -6)

minor axis = 4 units

$\frac{(y+3)^2}{21} - \frac{(x+1)^2}{4} = 1$



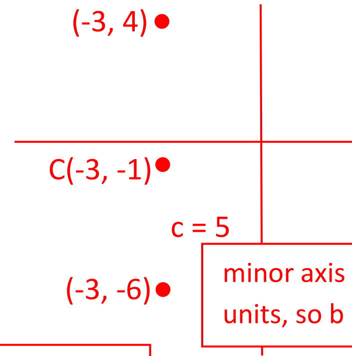
$c^2 = a^2 + b^2$
$25 = 9 + b^2$
$16 = b^2$
$b = 4$



Asymptote slopes: ±b/a, but we know a = 4, so

$$\frac{b}{a} = \frac{b}{4} = \frac{3}{2}$$

thus, b = 6



minor axis = 4 units, so b = 2

$c^2 = a^2 + b^2$
$25 = a^2 + 4$
$21 = a^2$

WHOLE 9-YARDS!

Find the center, "a", "b", and "c", determine if the graph's major axis is horizontal or vertical, sketch it, then name the vertices and foci.

8) $x^2 - y^2 - 6x + 4y = 20$

$x^2 - 6x + \_\_\_\_ - y^2 + 4y = 20 + \_\_\_\_$
$x^2 - 6x + 9 - y^2 + 4y = 20 + 9$
$(x - 3)^2 - y^2 + 4y = 29$
$-(x - 3)^2 + y^2 - 4y + \_\_\_\_ = -29 + \_\_\_\_$
$-(x - 3)^2 + y^2 - 4y + 4 = -29 + 4$
$-(x - 3)^2 + (y - 2)^2 = -25$
$(x - 3)^2 - (y - 2)^2 = 25$

$$\frac{(x-3)^2}{25} - \frac{(y-2)^2}{25} = 1$$

Center = (3, 2)

a = 5

b = 5

c = $5\sqrt{2}$

$c^2 = a^2 + b^2$
$c^2 = 25 + 25$
$c^2 = 50$
$c = 5\sqrt{2}$

Major axis: Horiz.

Vertices = (-2, 2) & (8, 2)

Foci = $(3 - 5\sqrt{2}, 2)$ & $(3 + 5\sqrt{2}, 2)$