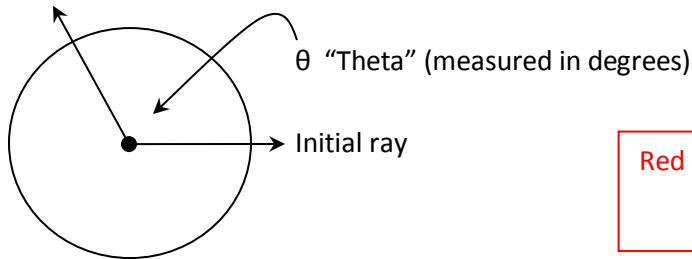


# Measurement of Angles

Terminal ray



Red

*Degree breakdown:  
Minutes & Seconds*

Convert the following degree measurements to decimal form. Round to nearest hundredth.

Examples: 1)  $50^{\circ}45'10''$   
50 degrees, 45 minutes, 10 seconds

$$= 50 + (45/60) + (10/3600)$$

punch in calc., round to 2-decimals

$$= 50.75^{\circ}$$

2)  $185^{\circ}5'32''$   
185 degrees, 5 minutes, 32 seconds

$$= 185 + (5/60) + (32/3600)$$

$$= 185.09^{\circ}$$

Your turn: 3)  $296^{\circ}30'15''$

$$= 296 + (30/60) + (15/3600)$$

$$= 296.50^{\circ}$$

4)  $11^{\circ}8'59''$

$$= 11 + (8/60) + (59/3600)$$

$$= 11.15^{\circ}$$

Convert the following decimals into degree-minute-second notation.

Examples: 1)  $42.7^{\circ}$

$$= 42 + (.7)(60)$$

$$= 42^{\circ} 42'$$

2)  $322.815^{\circ}$

$$= 322 + (.815)(60)$$

= 322 and 48.9 minutes

multiply .9 by (60) to get the seconds

$$= 322^{\circ} 48' 54''$$

Your turn: 3)  $199.25^{\circ}$

$$= 199 + (.25)(60)$$

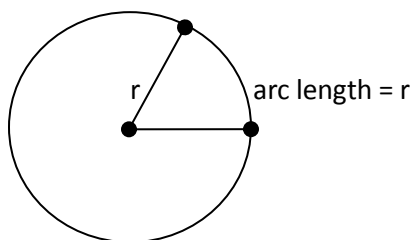
$$= 199^{\circ} 15'$$

4)  $-87.205^{\circ}$

Best to just include the -87 for the final answer.  $(.205)(60) = 12.3$

$$(.3)(60) = 18$$

Answer:  $-87^{\circ} 12' 18''$

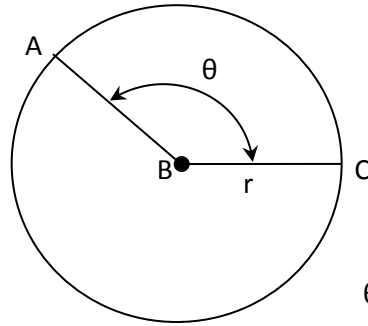


*1 radian unit is the equivalent measure of an angle in degrees, where the radius of the circle is equal to the arc length that it cuts.*

Determining the radian measure of a central angle.

Formula:

$$\theta = \frac{s}{r} \quad \text{needed in Section 7-2}$$



The distance from A to C is the arc-length noted with the variable "s"

θ is also known as the central angle

Converting degree measurements to radians

Change each degree measurement into radians (round answers to the nearest hundredth)

Examples: 1) 212°

$$212 \left( \frac{\pi}{180} \right) \text{ use a calculator...} \\ \approx 3.70 \text{ radians}$$

2) 18°6'

$$\left( 18 + \frac{6}{60} \right) \left( \frac{\pi}{180} \right) \\ \approx 0.32 \text{ radians}$$

Formula:

$$d \cdot \frac{\pi}{180}$$

Your turn: 3) 307°

$$307 \left( \frac{\pi}{180} \right) \text{ Use a calculator...} \\ \approx 5.36 \text{ radians}$$

4) 111°55'30"

$$\left( 111 + \frac{55}{60} + \frac{30}{3600} \right) \left( \frac{\pi}{180} \right) \\ \approx 1.95 \text{ radians}$$

Change each degree measurement into radians (in terms of π)

Examples: 1) 160°

$$160 \left( \frac{\pi}{180} \right) = \left( \frac{160}{1} \right) \left( \frac{\pi}{180} \right) = \frac{160\pi}{180} \\ \text{Reduce the fraction, leave pi.} \\ = \frac{8\pi}{9} \text{ radians}$$

2) 840°

$$840 \left( \frac{\pi}{180} \right) = \frac{840\pi}{180} \\ = \frac{14\pi}{3}$$

Formula:

$$d \cdot \frac{\pi}{180}$$

Your turn: 3) 45°

$$45 \left( \frac{\pi}{180} \right) = \frac{45\pi}{180} \\ = \frac{\pi}{4}$$

4) 480°

$$480 \left( \frac{\pi}{180} \right) = \frac{480\pi}{180} \\ = \frac{8\pi}{3}$$

## Converting radian measurements to degrees

Change each radian measurement (in either decimal or  $\pi$ -form) into degrees. Round any decimal answers to nearest hundredth.

Examples: 1)  $\frac{5\pi}{4}$

$$\left(\frac{5\pi}{4}\right)\left(\frac{180}{\pi}\right) \text{ Use a calculator.} \\ = 225^\circ$$

2) -9.5

$$-9.5\left(\frac{180}{\pi}\right) \\ \approx -544.31^\circ$$

Formula:

$$r \cdot \frac{180}{\pi}$$

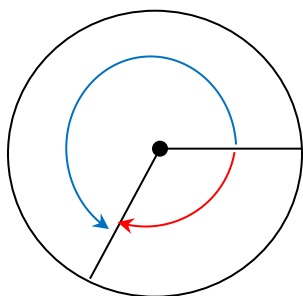
Your turn: 3)  $\frac{7\pi}{2}$

$$\left(\frac{7\pi}{2}\right)\left(\frac{180}{\pi}\right) \text{ Use a calculator.} \\ = 630^\circ$$

4) 1.06

$$1.06\left(\frac{180}{\pi}\right) \\ \approx 60.73^\circ$$

## Coterminal angles



If the blue angle is  $260^\circ$ , what is the red angle?

Are there any other angles that begin on the "Initial ray" and end at the same "Terminal ray"?

Yes, every time you rotate  $360^\circ$  in either direction, you end up at the same terminal ray.

Find two angles, one positive and one negative, which are coterminal to the given angle.

Examples: 1)  $135^\circ$

$$\begin{array}{r} 135 \\ +360 \\ \hline 495^\circ \end{array} \quad \begin{array}{r} 135 \\ -360 \\ \hline -225^\circ \end{array}$$

etc., you may continue to add or subtr. 360 to find more possibilities.

2)  $-513^\circ$

$$\begin{array}{r} -513 \\ +360 \\ \hline -153^\circ \end{array}$$

$\leftarrow$  could be used as neg. angle.

$$\begin{array}{r} +360 \\ 207^\circ \end{array}$$

add/subtr 360 for more

3)  $276^\circ 11'$

Method1  
Convert to decimal  
 $276 + \frac{11}{60}$ , then  
add/subtr 360  
 $= 636.18\bar{3}^\circ$   
and  $= -83.81\bar{6}^\circ$

Method2  
(add 360)  
 $276^\circ 11'$   
 $+ 360$   
 $636^\circ 11'$   
(subtr 360)  
 $360 \rightarrow$  borrow  $359^\circ 60''$   
 $- 276^\circ 11'' \quad - 276^\circ 11''$   
 $\hline 83^\circ 49''$   
Just add the sign  $-83^\circ 49''$