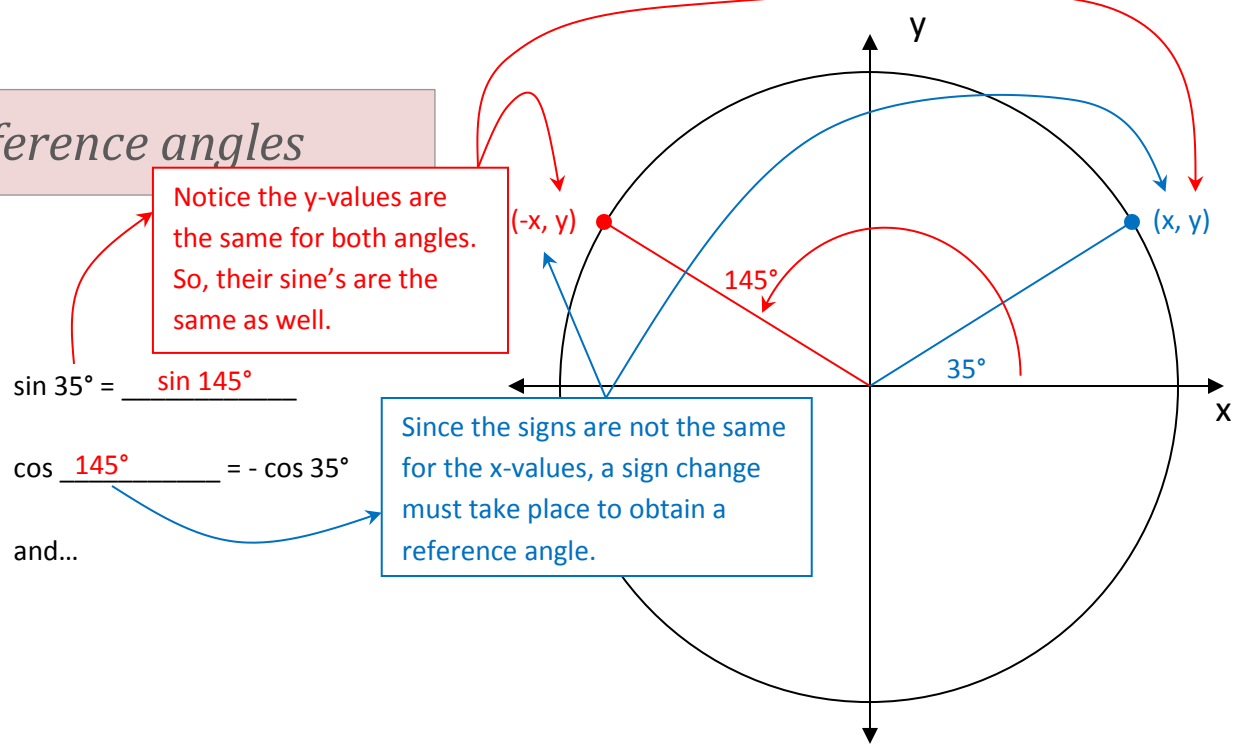


# EVALUATING & GRAPHING SIN & COS

## Reference angles



## Reference angle formulas

- (I)  $180^\circ - \alpha$  or  $\alpha - 180$  or  $360 - \alpha$   
(whichever one gets you into the first quadrant)
- (II) test the sign

Only had room for #1 and #2, sorry.

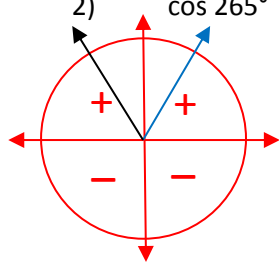
Examples: Name an (acute) reference angle for each angle given.

1)  $\sin 110^\circ$

2)  $\cos 265^\circ$

3)  $\cos 302^\circ$

#1)  $180 - 110 = 70$ ,  $70^\circ$  is in quad I  
So  $\sin 110^\circ = \sin 70^\circ$ ?? Use the "sin - sign" chart to determine if a sign change is necessary.



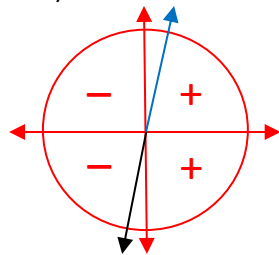
Since both angles end in a quadrant with positive y-values, no sign change would take place. Therefore:  $\sin 110^\circ = \sin 70^\circ$

4)  $\sin 444^\circ$

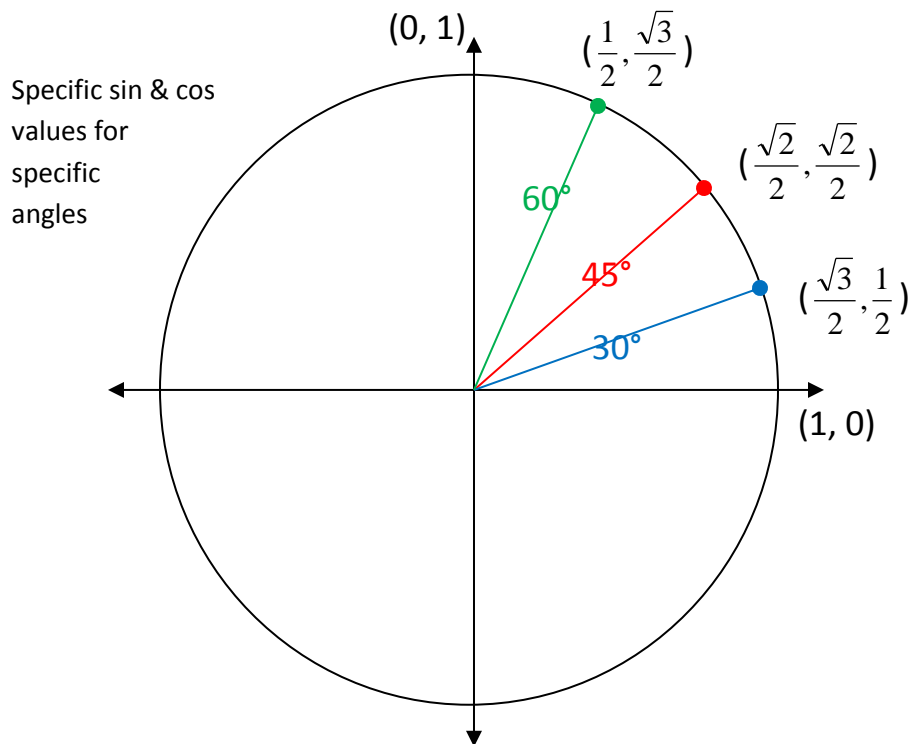
5)  $\sin 705^\circ$

6)  $\cos (-207^\circ)$

#2)  $180 - 265 \neq$  quad I  
 $265 - 180 = 85$  which is in quad I.  
So,  $\cos 265^\circ = \cos 85^\circ$ ?? Use the "cos - sign" chart to determine if a sign change is necessary.



Since the two angles have opposite x-value signs, a sign change must take place. Therefore:  $\cos 265^\circ = -\cos 85^\circ$



Give the exact values of each expression using reference angles.

1)  $\cos 240^\circ$

$240 - 180 = 60$  (reference  $\angle$ )  
 $\cos 240^\circ = \cos 60^\circ$ , but signs are different, so  $\cos 240^\circ = -\cos 60^\circ$   
 x-value for  $60^\circ = 1/2$  so  $-\cos 60^\circ = -1/2$ . And...  $\cos 240^\circ = -1/2$

2)  $\sin 135^\circ$

$180 - 135 = 45$  (reference  $\angle$ )  
 $\sin 135^\circ = \sin 45^\circ$ , since the signs are the same, no sign change.  
 y-value for  $45^\circ = \sqrt{2}/2$  so  
 $\sin 135^\circ = \sqrt{2}/2$  also.

3)  $\sin 180^\circ$

Since the coordinates on the unit circle are literally  $(-1, 0)$  for  $180^\circ$ , there is no need for a reference angle.  
 $\sin 180^\circ = 0$

4)  $\cos(-60^\circ)$

$-60 + 360 = 300$  (coterminal  $\angle$ )  
 $360 - 300 = 60$  (reference  $\angle$ )  
 $\cos(-60^\circ) = \cos 60^\circ$ . Since no sign change, x-value =  $1/2$ , and  $\cos(-60^\circ) = 1/2$  also.

5)  $\cos \frac{3\pi}{4}$

$\cos \frac{3\pi}{4} = \cos 135^\circ$   
 $180 - 135 = 45$  (reference  $\angle$ )  
 Signs for  $\cos 135^\circ$  and  $\cos 45^\circ$  are different so  $\cos 135^\circ = -\cos 45^\circ$ .  
 Therefore  $\cos \frac{3\pi}{4} = -\sqrt{2}/2$

6)  $\sin \frac{5\pi}{3}$

$\sin \frac{5\pi}{3} = \sin 300^\circ$   
 $360 - 300 = 60$  (reference  $\angle$ )  
 Signs for  $\sin 300^\circ$  and  $\sin 60^\circ$  are different so  $\sin 300^\circ = -\sin 60^\circ$ .  
 Therefore  $\sin \frac{5\pi}{3} = -\sqrt{3}/2$

7)  $\cos 7\pi$

$\cos 7\pi = \cos \pi$ . Since the coordinates on the unit circle are  $(-1, 0)$  for  $\pi$ , there is no need for a reference angle.  
 $\cos 7\pi = -1$ .

8)  $\sin \frac{5\pi}{6}$

$\cos \frac{5\pi}{6} = \cos 150^\circ$   
 $180 - 150 = 30$  (reference  $\angle$ )  
 Signs for  $\sin 150^\circ$  and  $\sin 30^\circ$  are the same so no sign change.  
 Therefore  $\sin \frac{5\pi}{6} = 1/2$

9)  $\cos 510^\circ$

$510 - 360 = 150$  (coterminal  $\angle$ )  
 $180 - 150 = 30$  (reference  $\angle$ )  
 Signs for  $\cos 150^\circ$  and  $\cos 30^\circ$  are different so  $\cos 150^\circ = -\cos 30^\circ$   
 Therefore  $\cos 150^\circ = -\sqrt{3}/2$