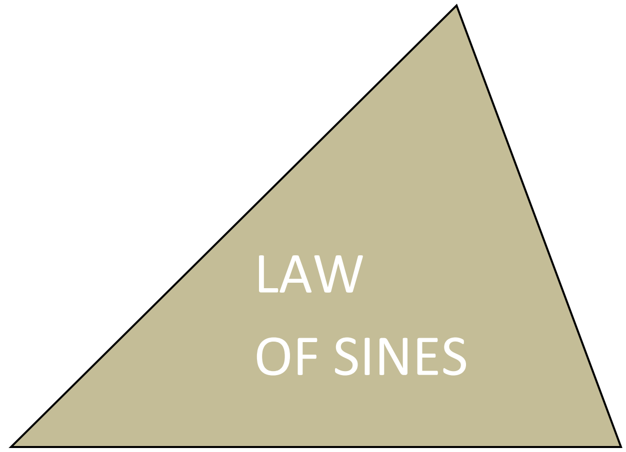
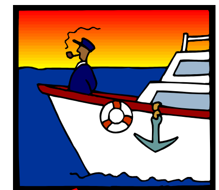
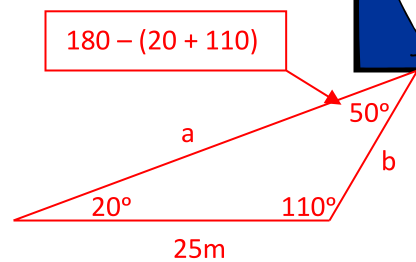
College Review Math
Section 9C

In ΔABC, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$





Example: During a recent drought, it has come to the attention of a river surveyor that there is a rock formation just below the surface of a frequently traveled river. While the water was so low the surveyor used a line between two telephone poles both located up river as reference angles. The closest pole had an angle of 110° in relation to the rocks, and the further 20°. If the distance between the poles was 25m, help the surveyor locate the rock formation so that a warning buoy may be placed there at a later date.



$\dfrac{\sin 110}{a} = \dfrac{\sin 50}{25}$ | $\dfrac{\sin 20}{b} = \dfrac{\sin 50}{25}$

asin50 = 25sin110 | bsin50 = 25sin20

a = $\dfrac{25\sin 110}{\sin 50}$ | b = $\dfrac{25\sin 20}{\sin 50}$

a ≈ 30.7m | b ≈ 11.2m

Two lines with these two lengths can be tied to the posts then strung out on a boat. The spot where they meet is where the buoy goes.

Simpler Examples:
Find all missing measurements for ΔABC (solve the triangle).

1) ∠A = 13°, ∠B = 75°, b = 12

∠C = 180 – (75 + 12) = 93

$\dfrac{\sin 93}{c} = \dfrac{\sin 75}{12}$ | $\dfrac{\sin 13}{a} = \dfrac{\sin 75}{12}$

csin75 = 12sin93 | asin75 = 12sin13

c = $\dfrac{12\sin 93}{\sin 75}$ | a = $\dfrac{12\sin 13}{\sin 75}$

c ≈ 12.4 | a ≈ 2.8

2) ∠A = 28°, ∠C = 135°, a = 2.8

∠B = 180 – (28 + 135) = 17

$\dfrac{\sin 17}{b} = \dfrac{\sin 28}{2.8}$ | $\dfrac{\sin 135}{c} = \dfrac{\sin 28}{2.8}$

bsin28 = 2.8sin17 | csin28 = 2.8sin135

b = $\dfrac{2.8\sin 17}{\sin 28}$ | c = $\dfrac{2.8\sin 135}{\sin 28}$

b ≈ 1.7 | c ≈ 4.2

3) ∠B = 50°, b = 10, c = 8.5

$\dfrac{\sin C}{8.5} = \dfrac{\sin 50}{10}$ | $\dfrac{\sin 89.4}{a} = \dfrac{\sin 50}{10}$

10sinC = 8.5sin50 | asin50 = 10sin89.4

C = $\sin^{-1}\left(\dfrac{8.5\sin 50}{10}\right)$ | a = $\dfrac{10\sin 89.4}{\sin 50}$

∠C ≈ 40.6° | a ≈ 13.1

∠A = 180 – (50 + 40.6) = 89.4°

4) ∠C = 6°, a = 29, c = 4

$\dfrac{\sin A}{29} = \dfrac{\sin 6}{4}$ | $\dfrac{\sin 124.7}{b} = \dfrac{\sin 6}{4}$

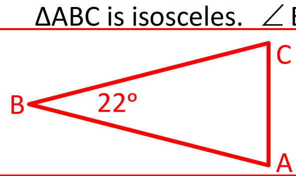
4sinA = 29sin6 | bsin6 = 4sin124.7

A = $\sin^{-1}\left(\dfrac{29\sin 6}{4}\right)$ | b = $\dfrac{4\sin 124.7}{\sin 6}$

∠A ≈ 49.3° | b ≈ 31.5

∠B = 180 – (6 + 49.3) = 124.7°

5) ΔABC is isosceles. ∠B = 22° and is not a base angle. b = 33.



∠A = ∠C = $\dfrac{180-22}{2}$

∠A = ∠C = 79°

$\dfrac{\sin 79}{a} = \dfrac{\sin 22}{33}$

asin22 = 33sin79

a = $\dfrac{33\sin 79}{\sin 22}$

a = c ≈ 86.5°

Follow up to Section 9-3

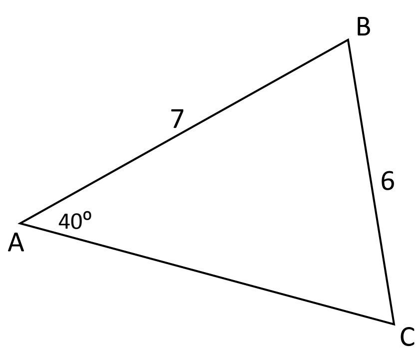
I. No solution (or no triangle possible)

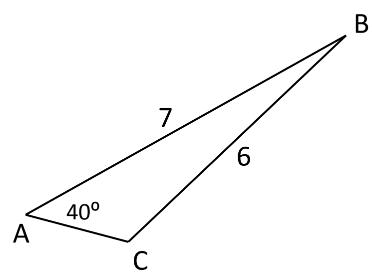
Construct the following triangle: ∠A = 40^o, a = 6, b = 5, c = 13

Computing an answer for angle B yields an answer of 32.4°. However, using the sin law, angle C reveals an error. Here is the reason: the two smaller sides of any triangle must total to more than the 3rd side. In this example, 6 + 5 is not greater than 13. Therefore, the triangle is impossible to construct in the first place.

II. More than one possible solution??

Construct this triangle: ∠A = 40^o, a = 6, c = 7. Which picture is the correct representation?





$\frac{sinC}{7} = \frac{sin40}{6}$

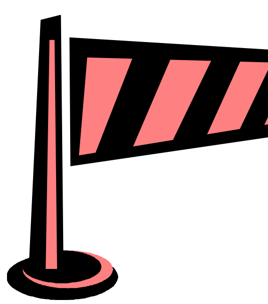
6sinC = 7sin40

sinC = $\frac{7sin40}{6}$

C = $\sin^{-1}(\frac{7sin40}{6})$

∠C ≈ 48.6

∠B ≈ 180 – (40 + 48.6) ≈ 91.4°



$\frac{sinC}{7} = \frac{sin40}{6}$

Setting up the sin law for this version yields the same equation. However, it is obvious from the drawing that angle C is obtuse in this picture, but acute in the other. Both scenarios are possible. DOWN SIDE OF THE SIN LAW: given two sides, and only one angle, there are two possible triangles that can be constructed; one acute, and one obtuse. The sin law will only give you one. How do you get the other answer? Simple; just subtract from 180. So, angle C in the above drawing:

∠C ≈ 180 – 48.6 ≈ 131.4°

∠B ≈ 180 – (40 + 131.4) ≈ 8.6°