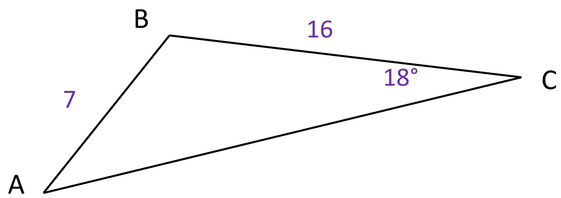
College Review
Section 9D

Law of Cosines

Warm up problem:

1) How can you find the missing side of the triangle shown below?



Law of Sines

$\frac{\sin 18}{7} = \frac{\sin A}{16}$

16sin18 = 7sinA

sinA = $\frac{16\sin 18}{7}$

A = $\sin^{-1}\left(\frac{16\sin 18}{7}\right)$

A ≈ 44.9°

∠A = 44.9°, ∠B = 117.1°

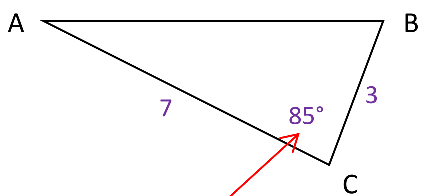
$\frac{\sin 18}{7} = \frac{\sin 117.1}{b}$

bsin18 = 7sin117.1

b = $\frac{7\sin 117.1}{\sin 18}$

b ≈ 20.2

2) Find the missing side of the triangle.



included angle = an angle between two given sides.

Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (3)^2 + (7)^2 - 2(3)(7)\cos 85$

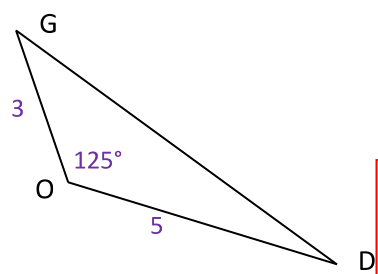
$c^2 = 9 + 49 - 42\cos 85$

$c^2 = 58 - 42\cos 85$

$c = \sqrt{58 - 42\cos 85}$ c ≈ 7.4

Examples (two sides & one included angle)
Solve each triangle.

3)



$o^2 = (3)^2 + (5)^2 - 2(3)(5)\cos 125$

$o^2 = 9 + 25 - 30\cos 125$

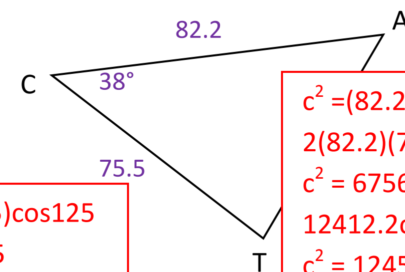
$o^2 = 34 - 30\cos 125$

$o = \sqrt{34 - 30\cos 125}$

o ≈ 7.2

Use the sine law to find a missing angle, subtract from 180 to get the third angle.

4)



$c^2 = (82.2)^2 + (75.5)^2 - 2(82.2)(75.5)\cos 38$

$c^2 = 6756.84 + 5700.25 - 12412.2\cos 38$

$c^2 = 12457.09 - 12412.2\cos 38$

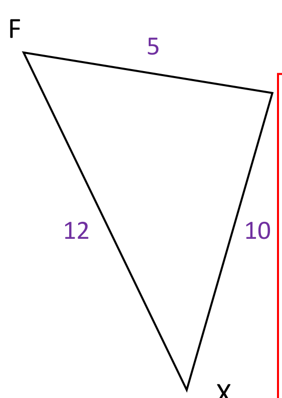
$c = \sqrt{above}$

c ≈ 51.7

repeat process from #3 for remaining measures.

Examples (three sides & no angle)
Solve each triangle.

5)



$10^2 = 5^2 + 12^2 - 2(5)(12)\cos F$

100 = 25 + 144 – 120cosF

100 = 169 – 120cosF

-69 = -120cosF

69/120 = cosF

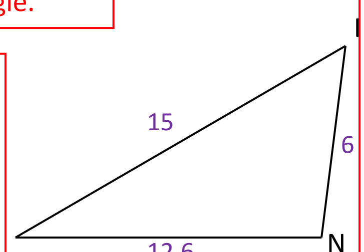
$F = \cos^{-1}(69/120)$

F ≈ 54.9°, repeat process for O

O ≈ 101°

X ≈ 180 – (54.9 + 101) ≈ 24.1°

6)



$6^2 = 15^2 + 12.6^2 - 2(15)(12.6)\cos H$

36 = 225 + 158.76 – 378cosH

36 = 383.76 – 378cosH

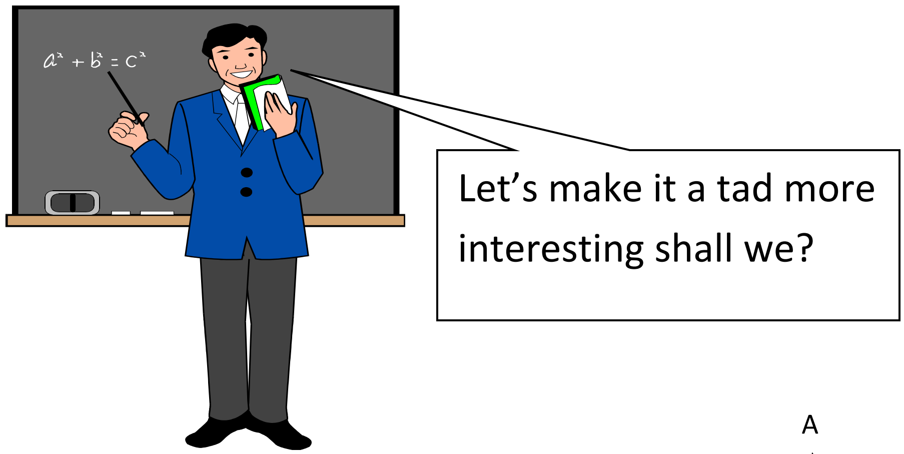
-347.76 = -378cosH

347.76/378 = cosH

$H = \cos^{-1}(347.76/378)$

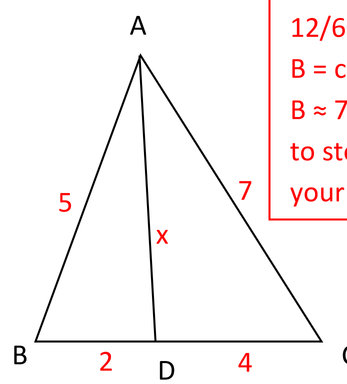
H ≈ 23.1°

Repeat for E ≈ 55.4°, N ≈ 180 – (23.1+55.4) ≈101.5°



Portions of triangles.

7) In the diagram at the right, AB = 5, BD = 2, DC = 4 and CA = 7. Find AD



Step 1: In order to find AD, we will need to know one of the angles.

$7^2 = 5^2 + (2+4)^2 - 2(5)(2+4)\cos B$

$49 = 25 + 36 - 2(5)(6)\cos B$

$49 = 61 - 60\cos B$

$-12 = -60\cos B$

$12/60 = \cos B$

$B = \cos^{-1}(12/60)$

$B \approx 78.5°$ *see Mr. Paull as to how to store the non-rounded value in your calculator.

Step 2: Use ΔABD to find AD.

$x^2 = 5^2 + 2^2 - 2(5)(2)\cos 78.5$

$x^2 = 25 + 4 - 20\cos 78.5$

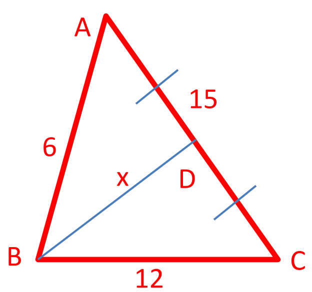
$x^2 = 29 - 20\cos 78.5$

$x = \sqrt{29 - 20\cos 78.5}$

$x = 5$

Medians of triangles.

8) Draw ΔABC with AB = 6, BC = 12, AC = 15. Draw and find the length of the median from B.



Step 1: The median (in blue) BD, is shown in the drawing. To find BD, we will need to know an angle.

$12^2 = 6^2 + 15^2 - 2(6)(15)\cos A$

$144 = 36 + 225 - 180\cos A$

$144 = 261 - 180\cos A$

$-117 = -180\cos A$

$117/180 = \cos A$

$A = \cos^{-1}(117/180)$

$A \approx 49.5°$

Step 2: Use ΔABD to find BD. Since BD is a median, AD = 7.5

$x^2 = 6^2 + 7.5^2 - 2(6)(7.5)\cos 49.5$

$x^2 = 36 + 56.25 - 90\cos 49.5$

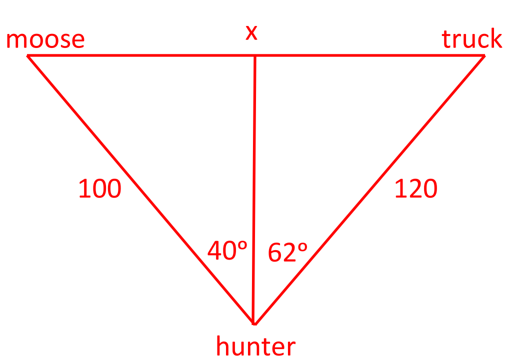
$x^2 = 92.25 - 90\cos 49.5$

$x = \sqrt{92.25 - 90\cos 49.5}$

$x \approx 5.8$

Word Problem example.

9) A hunter faces directly north. He notes that if he turns to the right at a 62° angle he can see his truck which is 120 feet away. He also notices to his left at an angle of 40° is a moose standing directly beneath his tree stand which he just left, and paced off 100 feet to the point at which he now stands. How far will he have to drag the moose to his truck after he pegs it with his high powered rifle?



angle from moose-truck = 40 + 62 = 102°

$x^2 = 100^2 + 120^2 - 2(100)(120)\cos 102°$

$x^2 = 10000 + 14400 - 24000\cos 102$

$x^2 = 24400 - 24000\cos 102$

$x = \sqrt{24400 - 24000\cos 102}$

$x \approx 171.4$ feet